Model for calculating stress intensity factors for an inclined crack at the leading edge of a gas turbine engine blade under the influence of centrifugal forces

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Abstract—The study is devoted to the issue of the destruction of a body with an inclined crack during rotation. A mathematical model for calculating stress intensity factors in a rectangular plate with an inclined crack under the influence of centrifugal tensile forces during rotation around an axis lying in the plane of the plate is constructed in the article. Based on the equations of the theory of elasticity and the principles of brittle fracture mechanics, relationships were obtained that relate the stress intensity factors of type I and II, the rotation speed and geometry of the plate, as well as the parameters of the crack: length, angle of inclination to the axis of rotation, distance from the axis of rotation to the crack. The complexity of the study is because the plate in question with an inclined crack is under the action of mass forces. Therefore, the values of the effective stresses are not the same along the crack edge. Accordingly, stress intensity factors will depend on the location of the crack relative to the axis of rotation. The influence of the crack location and plate rotation speed on the change in stress intensity factor values is analyzed based on the results obtained. As the distance from the axis of rotation to the crack increases, the values of the stress intensity factors decrease. As the plate rotation frequency increases, the stress intensity coefficients increase according to a parabolic law. The results of the study can be used to assess the limit state of the rotating blades of a gas turbine engine in the presence of an inclined crack. The mathematical model can find practical application for assessing the critical speed regime of blade rotation in the presence of cracks of various lengths and angles of inclination to the rotation axis.

Keywords — stress intensity factor, turbine blade, plate, inclined crack, centrifugal forces, rotation.

I. Introduction

Modern turbomachinery engineering is inextricably linked with such industries as aircraft manufacturing, shipbuilding, energy, etc. Every year new models of gas turbine engines appear in these industrial sectors. One of the most important elements of gas turbine engines are nozzles and rotor blades. The rotor blades of a gas turbine engine are extremely loaded elements, since they are exposed to centrifugal tensile forces, and bending gas forces and operate at high temperatures. The operation of many aircraft and, accordingly, flight safety depend on the good condition of turbine blades.

One of the most important defects in turbine blades are cracks, the formation and development of which can lead to failure of the engine and the entire aircraft. When the engine operates at high speeds, damage in the form of cracks is quite common in turbine blades. Features of the destruction of turbine blades with cracks, and issues of the propagation of fatigue cracks in the body of a turbine blade are reflected in works, [1], [2], [3]. Today, there are various approaches for diagnosing such damage using non-destructive testing, many of which are based on analyzing changes in the dynamic characteristics of a turbine blade in the presence of a crack. In works, [4], [5], [6], the authors considered a system for detecting damage in gas turbine blades based on capsules containing an ionizing substance placed in the body of the turbine blade. However, the successful implementation of this system requires solving several issues: determining the limit state at which crack growth begins; and calculating the trajectory of crack growth during loading.

Today, the assessment of the critical state of a body with a discontinuity is carried out by comparing various criterion values with limit values. Today, the stress intensity factor, J-integral, etc. are used as such parameters. The stress intensity factor is one of the main indicators for assessing the limited state of a body with a crack and predicting its destruction, which is used in problems of fracture mechanics. This indicator depends on the geometry of the body, the location and parameters of the crack, and the characteristics of force loading. Accordingly, there are various methods for determining it. Of course, experimental methods are the most preferable, but the experiments performed do not always reflect all the most important features of the process under study. Theoretical methods for estimating stress intensity factors, as a rule, are based on the application of the relations of the theory of elasticity.

Today, the problem of calculating stress intensity factors in bodies of complex geometry with a crack is very relevant. There are various approaches for determining stress intensity factors and assessing the stress state during crack development, which is reflected in works, [7], [8], [9]. Many problems today are solved using the finite element method. Finite element analysis methods make it possible to calculate stress intensity factors for various complex types of loading. The most universal is the compliance method, which requires estimating the potential energy of strain in the vicinity of a crack based on the finite element method. Despite the extensive research on the issue of calculating the stress intensity factor, there are still no exact methods for its calculation. Numerical mesh methods require the construction of a uniform and sufficiently fine mesh near the crack to obtain stress intensity factors. In addition, many finite element methods apply to solving direct problems to obtain quantitative results. However, today in many engineering problems it is necessary to establish functional relationships between the stress intensity factor and the acting loads. Therefore, despite all the advantages of finite element analysis methods, it is important to develop analytical methods for calculating stress intensity factors.

Most experimental and theoretical studies at the moment are devoted to the issues of propagation of tensile cracks. However, in recent years, more and more scientific works have been devoted to assessing the limiting state of cracks under mixed fracture mode conditions. In particular, the problem of the occurrence and growth of fatigue cracks under the simultaneous action of tensile and shear stains is considered in works, [10], [11], [12], [13], [14]. The problem of assessing the stress-strain state in bodies with an inclined crack was studied in [15], [16], [17], [18]. Many works are devoted to the development of cracks and assessment of the stress-strain state in bodies with a crack made of anisotropic materials and composite materials, which was considered by the authors in [12], [19], [20]. The difficulty in assessing the state of a body with an inclined crack lies in the fact that such a crack, as a rule, is a mixed type crack, which requires the calculation of several stress intensity factors for tensile and shear cracks, which significantly complicates the solution of the problem. One of the promising approaches to solve this issue is the method of equivalently replacing an inclined crack with a transverse crack, which is considered using the example of a beam in [21]. Experimental studies of fatigue fracture of bodies with an inclined crack for various alloys were considered in [16], [18], [22]. Modeling of fatigue crack growth was carried out by the authors in studies, [23], [24].

Few works are devoted to the development of cracks in bodies under the influence of centrifugal forces as a result of rotation. An example of such a body is the blade of a gas turbine engine, which, during the rotation of the rotor, is exposed to tensile mass forces that vary along the height of the turbine blade. Some modern researchers pay attention to the issue of calculating the stress intensity factor during the rotation of bodies with a crack. In most of these studies, a disk rotating relative to the diameter is considered a body with a crack, which is reflected in the works, [25], [26], [27], [28], [29], [30].

An analysis of the literature shows that most of the research on the subject of the article can be classified as experimental studies that are associated with conducting field experiments, in particular, to determine the fracture toughness of specific materials, as well as theoretical studies. Theoretical studies are mostly related to the use of CAE systems based on the finite element method for estimating the stress-strain state of bodies with cracks. At the same time, many finite element software packages are more widely used to calculate direct problems about the critical state of a body with a crack, when, under given loading conditions, the values of stress intensity coefficients are determined for a given geometry of a body with a crack. However, the inverse problems in fracture mechanics are relevant today, for example, when developing a system for diagnosing damage in turbine blades, [5], [6], [8], it is required to calculate the critical crack length and the critical rotation speed of a plate with a defect based on fracture viscosity data.

About gas turbine blades, one of the first works devoted to the issue of determining stress intensity factors is work, [31]. Estimation of the stress intensity factor is extremely important for optimizing the system for detecting damage in gas turbine blades, since it allows one to estimate the stress state of the turbine blade at which crack movement will occur.

The study of the stress-strain state of a turbine blade with a crack requires the use of fracture mechanics methods. The practical significance of such studies is because solving the problems of predicting the critical state of bodies with a crack will improve the operating efficiency of a gas turbine engine without emergencies. To calculate stress intensity factors, it is necessary to compile a system of equations that objectively reflect the stress state under existing loads. The complexity of the research lies in the fact that a turbine blade is a body of complex geometry. To date, there are no reliable relationships that allow taking into account all the geometric features of the turbine blade body and the parameters of force and thermal loading. Therefore, it is relevant to develop methods for calculating stress intensity factors for turbine blades with a crack, which would allow taking into account the effect of high-speed loading conditions and the location of the crack relative to the axis of rotation. These methods are especially important when solving inverse problems, in particular, for the development of damage detection systems, when, based on data on the fracture toughness of a material, it is necessary to calculate the critical length of a crack and the critical rotation speed of the blade. Some issues of estimating the stress-strain state of shell elements were considered in [32].

II. Problem statement

The blade of a gas turbine engine is subject to centrifugal tensile forces during rotation. For simplification, we will consider the model problem using the example of a thin-walled plate that approximates the body of a turbine blade. Thus, the purpose of the study was to develop a mathematical model for calculating the stress intensity factor in a plate with an inclined crack, taking into account the speed of rotation of the body, the distance from the axis of rotation to the crack, as well as the angle of inclination of the crack to the axis of rotation. To solve the research problem, the following assumptions and simplifications were made:

– The angular velocity of rotation of the plate remains constant, the axis of rotation lies in the plane of the plate;

– The effect of surface bending forces, due to their smallness compared to centrifugal forces, is not taken into account;

– The effect of temperature loads on the plate is not taken into account;

– plate is in a uniaxial stressed state as a result of the action of centrifugal forces during rotation;

– plate material is a linear elastic body, plastic strains are not detected;

– strains are small compared to the size of the body and the length of the crack;

– deviation of the crack growth trajectory in the plate from the initial direction is not considered.

III. Research methodology

Let us consider a rectangular plate of width b, height h, thickness δ, rotating relative to the axis Ox_1 passing through the lower boundary of the plate (Fig. 1). The inclined crack is localized on the left border of the plate. The stress state of the plate during rotation is determined based on the equilibrium equations and boundary conditions:

$$\sigma_{ij,j} + \rho X_i = 0, \tag{1}$$

$$\sigma_{ij} n_j = t_i^*|_\Gamma, \tag{2}$$

where σ_{ij} – components of the stress tensor, ρ – density, X_i – mass forces, n_j – components of the normal vector, t_i^* – stresses on the surface of the body with the boundary Γ.

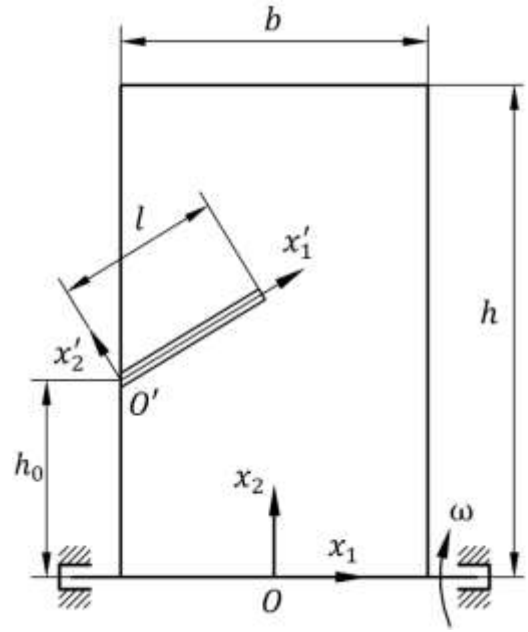

Fig. 1 Model of a rotating plate with an inclined crack

Let us introduce a coordinate system $\mathbf{x} = x_i \mathbf{e}_i, i = \overline{1;3}$ ($\mathbf{e}_i$ is an orthonormal basis) with the origin at point O. Consider an inclined crack of length l with the origin on the left side of the plate. The position of an inclined crack in the form of a rectangular cut is determined by the normal vector $\mathbf{n} = n_i \mathbf{e}_i$ to the surface of the crack bank:

$$\mathbf{n} \cdot \mathbf{e_1} = -\sin\alpha, \mathbf{n} \cdot \mathbf{e_2} = \cos\alpha, \mathbf{n} \cdot \mathbf{e_3} = \mathbf{0}, \tag{3}$$

where α is the angle of inclination of the crack line to the axis of rotation.

Let us introduce a local coordinate system $\mathbf{x}' = x'_i \boldsymbol{e}'_i$, $i = \overline{1;3}$ with the origin at point O', corresponding to the crack apex on the left edge of the plate. The transition from a coordinate system $\mathbf{x}$ to a new local coordinate system $\mathbf{x}'$ satisfies the transformations:

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1' \\ x_2' \\ x_3' \end{Bmatrix} + \begin{Bmatrix} -b/2 \\ h_0 \\ 0 \end{Bmatrix}, \tag{4}$$

where b – width of the plate, h_0 – distance from the axis of rotation to the point O'.

The plate is under the action of mass forces acting in the direction of the axis Ox_2. The stress state under the action of only centrifugal tensile forces will be determined according to the equilibrium equation (1):

$$\frac{\partial \sigma_{22}}{\partial x_2} - \rho\omega^2 x_2 = 0, \tag{5}$$

under boundary condition (2) at the peripheral section, where there are no stresses:

$$\sigma_{22}|_{x_2=h} = 0. \tag{6}$$

where ω – angular velocity of rotation.

The solution to the system of equations (5), (6) has the form:

$$\sigma_{22}(x_2) = \frac{1}{2}\rho\omega^2(h^2 - x_2^2). \quad (7)$$

Relationship (7) is consistent with the formula for radial stresses in a turbine blade in the absence of gas bending forces according to [33]. To solve the problem of rotation of a plate with an inclined crack, it is necessary to obtain a function that reflects the change in radial stresses in the local coordinate system $\mathbf{x}'$. Using transformations (3), (4), we present equation (7) in the form:

$$\sigma_{22}(x_1', x_2') = \frac{1}{2}\rho\omega^2(h^2 - [\sin\alpha\, x_1' + \cos\alpha\, x_2' + h_0]^2). \quad (8)$$

Since the tangent to the crack line in the plate is not orthogonal to the direction of the acting stresses, but makes an angle $0 < \alpha < 90^0$, then the inclined crack is a mixed type crack. Accordingly, it is required to determine the stress intensity factors $K_{\mathrm{I}} \neq 0$, $K_{\mathrm{II}} \neq 0$.

For this purpose, we apply the approach proposed in [34], according to which it is necessary to consider the equilibrium of a part of the plate after cutting in the direction of the crack line. Thus, let us cut the plate along the crack line and consider the equilibrium of the remaining part of the body. It should be taken into account that additional forces arise at the right end of the crack, [34]:

$$\int_0^{\Delta l_{\mathrm{II}}} \sigma_{r\theta}\, \delta dr \text{ и } \int_0^{\Delta l_{\mathrm{I}}} \sigma_{\theta\theta}\, \delta dr, \quad (9)$$

which compensate for the effects of forces not transmitted through the crack line, in projections onto the tangent and normal to the crack line:

$$\mathbf{F}^{(n)} \cdot \mathbf{e}_1' = \sin\alpha \int_0^l \sigma_{22}\,(x_1')\delta dx_1', \quad (10)$$

$$\mathbf{F}^{(n)} \cdot \mathbf{e}_2' = \cos\alpha \int_0^l \sigma_{22}\,(x_1')\delta dx_1{}', \quad (11)$$

where r, θ – variables of the cylindrical coordinate system with the origin at the right end of the crack, $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ – normal and shear stresses that arise in the vicinity of the right end of the crack at $r = \Delta l_{\mathrm{I}}$ and $r = \Delta l_{\mathrm{II}}$.

Equating the right-hand sides of relations (10), (11) with the corresponding forces (9) near the right crack tip, we obtain the following equations:

$$\cos\alpha \int_0^l \sigma_{22}\,(x_1')dx_1' = \int_0^{\Delta l_{\mathrm{I}}} \sigma_{\theta\theta}\, dr, \quad (12)$$

$$\sin\alpha \int_0^l \sigma_{22}\,(x_1')dx_1' = \int_0^{\Delta l_{\mathrm{II}}} \sigma_{r\theta}\, dr. \quad (13)$$

The unknown quantities Δl_{I} and Δl_{II} will be found from the conditions:

$$\sigma_{\theta\theta}\big|_{\substack{r=\Delta l_{\mathrm{I}} \\ \theta=0}} = (\mathbf{e_2} \cdot \mathbf{e}_2')\sigma_{22}\big|_{\substack{x_1'=l+\Delta l_{\mathrm{I}} \\ x_2'=0}}, \quad (14)$$

$$\sigma_{r\theta}\big|_{\substack{r=\Delta l_{\mathrm{II}} \\ \theta=0}} = (\mathbf{e_2} \cdot \mathbf{e}_1')\sigma_{22}\big|_{\substack{x_1'=l+\Delta l_{\mathrm{II}} \\ x_2'=0}}. \quad (15)$$

Considering that near the crack tip the following relations are satisfied according to [34]:

$$K_{\mathrm{I}} = \sigma_{\theta\theta}\sqrt{2\pi r},\ K_{\mathrm{II}} = \sigma_{r\theta}\sqrt{2\pi r}, \quad (16)$$

then, based on conditions (14) – (16), we obtain:

$$\Delta l_{\mathrm{I}} = \frac{K_I^2}{2\pi\cos^2\alpha\, \sigma_{22}^2\big|_{\substack{x_1'=l+\Delta l_{\mathrm{I}} \\ x_2'=0}}},\qquad \Delta l_{\mathrm{II}} = \frac{K_{II}^2}{2\pi\sin^2\alpha\, \sigma_{22}^2\big|_{\substack{x_1'=l+\Delta l_{\mathrm{II}} \\ x_2'=0}}} \quad (17)$$

Relations (12), (13) when integrating the right-hand sides of the equations are reduced, taking into account (8), (16), to the equations:

$$\cos\alpha \int_0^l \left[\frac{1}{2}\rho\omega^2(h^2 - [\sin\alpha\, x_1' + R]^2)\right] dx_1' = K_I\sqrt{\frac{\Delta l_{\mathrm{I}}}{2\pi}}, \quad (18)$$

$$\sin\alpha \int_0^l \left[\frac{1}{2}\rho\omega^2(h^2 - [\sin\alpha\, x_1' + R]^2)\right] dx_1' = K_{II}\sqrt{\frac{\Delta l_{\mathrm{II}}}{2\pi}}. \quad (19)$$

Integrating the left-hand sides of equations (18), (19) taking into account (8) and (17), we obtain $\Delta l_{\mathrm{I}} = \Delta l_{\mathrm{II}} = \Delta l$. Then the system of equations (18), (19) is reduced to solving the equation:

$$(h^2 - [(l + \Delta l)\sin\alpha + h_0]^2)\Delta l = lh^2 + \\ + \frac{1}{3\sin\alpha}[h_0^3 - (h_0 + \sin\alpha\, l)^3]. \quad (20)$$

The solution to equation (20) with respect to Δl can be easily found analytically or numerically. Then the stress intensity factors are determined according to equations (18), (19) in the form:

$$K_{\mathrm{I}} = \frac{1}{2}\rho\omega^2\cos\alpha\,[lh^2 + \frac{1}{3\sin\alpha} \cdot \\ \cdot [h_0^3 - (h_0 + \sin\alpha\, l)^3]]\sqrt{2\pi/\Delta l}, \quad (21)$$

$$K_{\mathrm{II}} = \frac{1}{2}\rho\omega^2\sin\alpha\,[lh^2 + \frac{1}{3\sin\alpha} \cdot \\ \cdot [h_0^3 - (h_0 + \sin\alpha\, l)^3]]\sqrt{2\pi/\Delta l}, \quad (22)$$

Equations (21), (22) allow one to calculate stress intensity factors for an inclined crack located at the boundary of a rotating plate.

IV. CALCULATION RESULTS

To analyze changes in stress intensity factors for various parameters of crack location and plate rotation conditions, we will carry out test calculations. Considering that the blades of different stages of a gas turbine engine have different sizes, the calculation will be carried out for such geometric parameters of the gas turbine engine blade at which the stress state in the blade will be maximum. For most real gas turbine engines, the maximum height of the blade body varies on average within 0.15 m, and the diameter of the gas turbine engine rotor can reach 0.7 m. Therefore, we will use these geometric parameters in the calculations. When a turbine blade rotates, the stress state is predominantly determined by the action of tensile centrifugal forces, which depend on the rotation speed. Therefore, we will carry out calculations at various angular speeds from 5 to 15 thousand revolutions per minute, which corresponds to the real operating conditions of gas turbine engine rotors. Initial data for calculation: $\rho = 7800\,\mathrm{kg/m^3}$, $h = 0.35$ m, $h_0^{\min} \le h_0 < h_0^{\max}$, $h_0^{\min} = 0.2$ m, $h_0^{\max} = 0.35$ m, $b = 0.1$ m.

When an inclined crack is located at an angle $\alpha = 45^0$ to rotation axis, the stress intensity factors (21), (22) will take the same values. The dependence of the stress intensity factors on the number of revolutions of rotation N of the plate for various options for the location of the crack along the height of the plate is presented in Fig. 2.

Considering that the stress state changes in the radial direction from maximum values in the region at $h_0 = h_0^{\min}$ to zero in the region at $h_0 = h_0^{\max}$, Fig. 3 shows the dependence of stress intensity factors on the relative lengths of cracks in the most loaded region. The dependence of the stress intensity coefficients on the angle of inclination of the crack to the axis of rotation in the region at $h_0 = h_0^{\min}$ is shown in Fig. 4, Fig. 5.

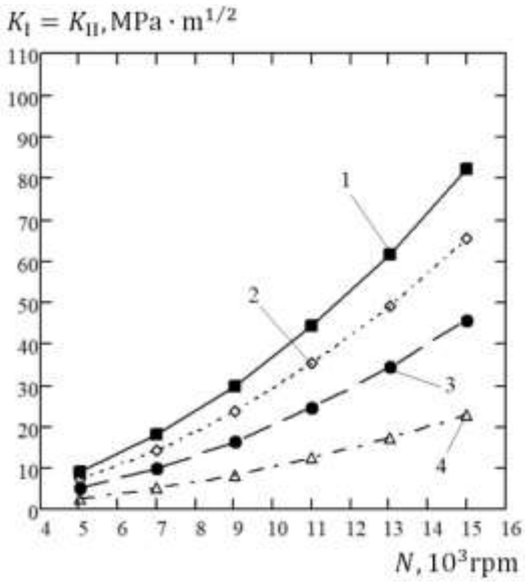

Fig. 2 Dependence of stress intensity factors $K_\mathrm{I} = K_\mathrm{II}$ for an inclined crack at $\alpha = 45^0$ on the rotational speed at various values $h_0 = \beta h_0^{\max} + (1-\beta) h_0^{\min}$: $1 - \beta = 0$, $2 - \beta = 0.25$, $3 - \beta = 0.5$, $4 - \beta = 0.75$

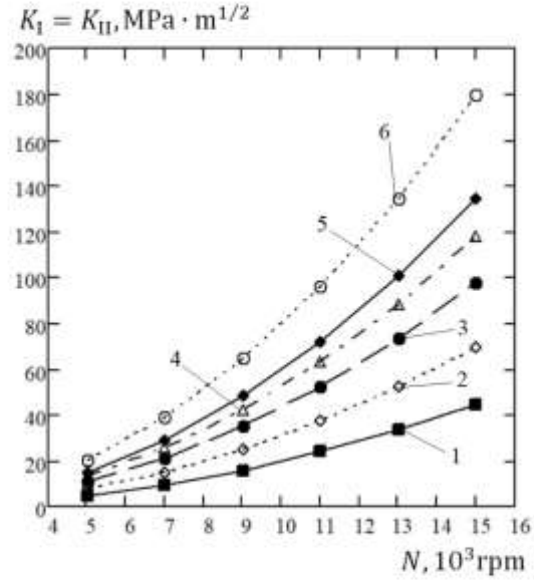

Fig. 3 Dependence of stress intensity factors $K_\mathrm{I} = K_\mathrm{II}$ for an inclined crack at $\alpha = 45^0$ at $h_0 = h_0^{\min}$ on the rotation speed for different relative crack lengths: $1 - l/b = 0.01$, $2 - l/b = 0.025$, $3 - l/b = 0.05$, $4 - l/b = 0.075$, $5 - l/b = 0.1$, $6 - l/b = 0.2$

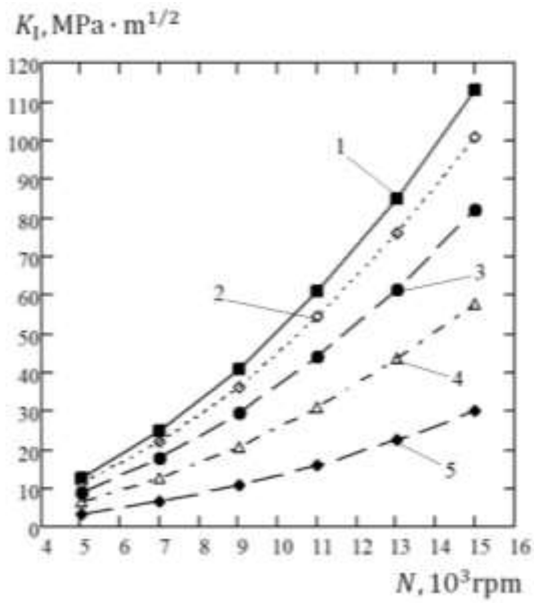

Fig. 4 Dependence of stress intensity factor K_I for an inclined crack at $h_0 = h_0^{\min}$ on the rotation speed for various initial angles of inclination of the crack α to the axis of rotation: $1 - \alpha = 15^0$, $2 - \alpha = 30^0$, $3 - \alpha = 45^0$, $4 - \alpha = 60^0$, $5 - \alpha = 75^0$

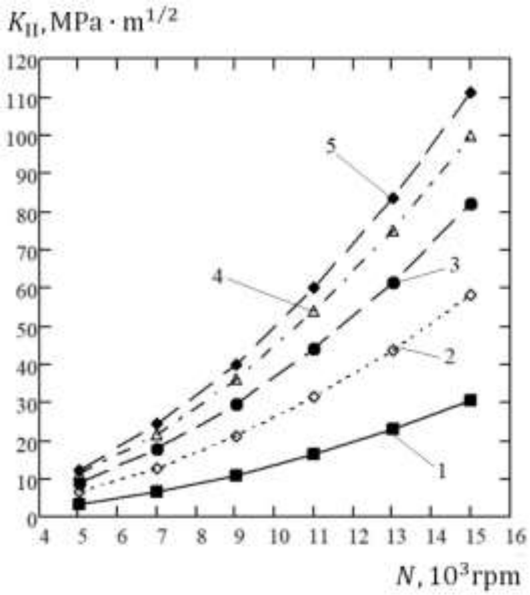

Fig. 5 Dependence of stress intensity factor K_{II} for an inclined crack at $h_0 = h_0^{\min}$ on the rotation speed for various initial angles of inclination of the crack α to the axis of rotation: $1-\alpha = 15^0,\ 2-\alpha = 30^0,\ 3-\alpha = 45^0,\ 4-\alpha = 60^0,\ 5-\alpha = 75^0$

V. Discussion of the results

According to the calculation results presented in Fig. 2, when an inclined crack is located at an angle of 45^0 to the plate rotation axis, the stress intensity factors of type I and type II are equal. The results obtained demonstrate that the distance from the axis of rotation to the crack h_0 affects the values of the stress intensity factors K_I and K_{II}. As the distance from the axis of rotation to the crack increases, the values of the stress intensity factors decrease, reaching the highest values in the region of the root section at $h_0 = h_0^{\min}$. Consequently, the most dangerous are cracks located in the area of the root section of the blade, since the values of the stress intensity factors in this case can be the greatest. All other things being equal, in the root section there is a higher probability of accelerated growth of cracks in comparison with the peripheral region farthest from the axis of rotation.

With an increase in the angular velocity of rotation, the stress intensity factors increase according to a parabolic law. According to the calculations in Fig. 3, as the crack length l increases, the values of the stress intensity factors increase.

At angles of inclination of the crack to the rotation axis other than 45^0, the stress intensity factors of types I and II are different (Fig. 4). At angles of inclination of the crack to the rotation axis $\alpha > 45^0$, there is a relationship between the stress intensity factors of types I and II: $0 < K_{\mathrm{I}}/K_{\mathrm{II}} < 1$. As the angle α increases, the ratio $K_{\mathrm{I}}/K_{\mathrm{II}}$ decreases; therefore, the edges of the crack will shift to a greater extent in the crack plane compared to the displacement in the direction normal to the crack plane. At angles of inclination of the crack to the axis of rotation $\alpha < 45^0$, the relationship between the stress intensity factors will take the form: $K_{\mathrm{I}}/K_{\mathrm{II}} > 1$, i.e. When the angle of inclination of the crack to the axis of rotation decreases, the stress intensity factor of type I has a predominant value for assessing the critical state.

It should also be noted that dependencies (21) and (22) obtained in the study, which establish the relationship between stress intensity factors, inclined crack parameters and speed loading conditions, can be used for practical purposes to determine the critical length of a crack at which its accelerated growth begins. For most materials, fracture toughness does not exceed 100 … 150 $\mathrm{MPa \cdot m^{1/2}}$. For example, for the material Inconel 718 with fracture toughness $K_{\mathrm{Ic}} = 73 \dots 87\ \mathrm{MPa \cdot m^{1/2}}$, [35], the limit state for a crack with an initial relative length $l/b = 0.2$according to Fig. 3 is achieved at a rotation speed of: 10 thousand revolutions per minute. As the crack length decreases, the value of the maximum rotation speed corresponding to the beginning of crack growth increases.

VI. Conclusion

The constructed mathematical model based on the equations of elasticity theory makes it possible to analytically calculate the stress intensity factors of types I and II for an inclined crack in a plate that is exposed to centrifugal tensile forces. According to the results of the study, the influence of the location of an inclined crack on the values of stress intensity factors relative to the axis of rotation was established:

– as the distance from the axis of rotation of the plate to the crack increases, the values of the stress intensity factors decrease.

– as the angle of inclination of the crack α to the axis of rotation increases, the ratio $K_{\mathrm{I}}/K_{\mathrm{II}}$ decreases. For angles $\alpha > 45^0$, the predominant value for assessing the critical state of a plate with an inclined crack will be the stress intensity factor of type II, and for angles $0 < \alpha < 45^0$ – the stress intensity factor of type I. At angle $\alpha = 45^0$ the equality $K_{\mathrm{I}} = K_{\mathrm{II}}$ is satisfied.

The influence of the plate rotation speed on the values of the stress intensity factors is also analyzed: with increasing rotation speed, the values of the stress intensity factors increase according to a parabolic law.

The obtained relationships make it possible to establish the relationship between stress intensity factors, plate geometry, its rotation speed, as well as crack parameters: length, distance to the rotation axis and angle of inclination to the rotation axis. The constructed mathematical model can be used to solve inverse problems of fracture mechanics. The research results can find practical application in the field of turbomachinery for assessing the critical state of turbine blades under the influence of centrifugal tensile forces, in the development of damage detection systems for turbine blades of a gas turbine engine, where it is necessary to determine the critical lengths of cracks in various sections of the turbine blade body under various high-speed loading conditions based on fracture toughness data.

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- Ivan K. Andrianov carried out the formulation of the problem research and modelling.
- Elena K. Chepurnova carried out a series of calculations.

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Conflict of Interest

The authors have no conflict of interest to declare that is relevant to the content of this article.