Modelling initial geometric imperfections of steel plane frames using entropy and eigenmodes

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Abstract— The article introduces an innovative approach to modelling initial geometric imperfections in steel plane frames. Initial imperfections are introduced using the analysis of normalised deformations of elastic buckling modes. The scale of these modes is assessed by applying Shannon entropy and potential energy analysis. The presented case study demonstrates a decreasing scale of the elastic buckling modes. The entropy computed from the deformation reveals a new perspective on buckling modes and provides a more profound understanding of steel frame behaviour. The case study results indicate that anti-symmetric buckling modes exhibit higher entropy than symmetric buckling modes. This entropy-based analysis enables the differentiation between symmetric and anti-symmetric buckling modes, which is particularly valuable when the critical buckling loads of sway and non-sway buckling modes are closely aligned or overlap.

Keywords—Buckling, entropy, eigenmode, frame, structure, elastic stability, imperfections.

I. Introduction

Steel plane frames commonly exhibit imperfections in their straightness, resulting from manufacturing and erection tolerances, [1]. These initial geometric imperfections can significantly impact the resistance and stiffness of steel frames, encompassing imperfections such as frame out-of-plumbness, member out-of-straightness (bow), and local cross-sectional imperfections. In global frame analysis, it is often customary to assume the worst-case scenario for the pattern of initial imperfections, aiming to maximize their destabilizing effects under applied loads. However, this approach tends to be excessively conservative.

In reality, both initial out-of-straightness (bow) and out-of-plumbness (sway) are random in nature, [2], [3]. However, due to the unique characteristics of each steel structure, it is impractical to measure and statistically evaluate these imperfections across a large number of identical frames, [4]. Consequently, alternative empirical methods based on standards or experience are employed, [5].

In advanced structural analyses, several widely used methods are employed to model initial imperfections. These methods include adjusting nodal coordinates, introducing notional horizontal forces, reducing the stiffness of members, and superimposing scaled elastic buckling modes, [6]. Typically, the first buckling eigenmode is utilized to represent the initial imperfections, assuming it to be the most critical, [7], [8]. However, it should be noted that the actual collapse mode of a structure, particularly for frame structures, may not necessarily align with the first buckling mode.

In [9], the author demonstrated that when the critical buckling loads of two distinct buckling modes coincide, the sensitivity to imperfections increases significantly. Specifically, there is a specific critical bracing stiffness for which the critical buckling loads of non-sway (symmetric) and sway (anti-symmetric) buckling modes coincide, [9]. A complete coincidence of critical forces makes distinguishing between the first and second buckling modes impossible, intensifying the sensitivity to initial imperfections, [10].

To address the challenges in determining initial imperfections, some researchers, [11], propose incorporating the first couple of eigenmodes in the modelling process. This approach ensures that the shape closely aligns with the validated recommendations found in design standards, [7].

The primary focus of this article lies in the methodology of superimposing scaled elastic buckling modes using the potential energy of deformation and introducing entropy as a new measure. Entropy provides a non-conventional perspective on buckling modes, enabling differentiation between the deformations of non-sway (symmetric) and sway (anti-symmetric) buckling modes. The article introduces a new methodology for generating initial imperfections by superimposing scaled elastic buckling modes.

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II. Buckling and Entropy

In the context of buckling, a slender structure initially possesses elastic stability, where all nodes are arranged in an initial configuration. As the compressive load on the structure increases, its stability diminishes. This decrease in elastic stability is attributed to the bending stiffness of the members, which progressively allows the nodes to deviate from their initial positions. Eventually, the bending stiffness reaches zero, marking the point at which buckling occurs. Mathematically, this corresponds to the determinant of the stiffness matrix becoming zero.

The eigenmode shapes of steel plane frames represent the natural deformations that the frame can exhibit when the determinant of the stiffness matrix reaches zero. These critical points can be reached at various load values.

A. Eigenmodes and Entropy

Each eigenmode can be characterized by a specific number of sine waves within its shape. As we move to higher-order eigenmodes, the complexity of the deformation patterns increases. Successive modes introduce additional nodes or points of zero displacements along the length of the frame. These nodes effectively divide the frame into smaller segments, forming more intricate deformation patterns.

Let's consider a simple example of a plane frame composed of two columns connected by a cross-beam, see Fig. 1.

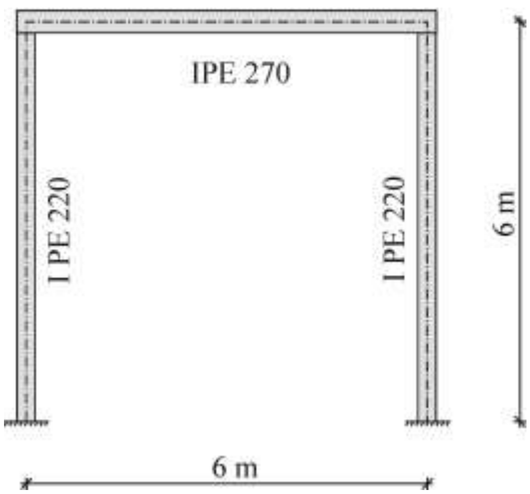

Fig. 1 The example of a steel plane frame with slender columns.

In the numerical example, the steel frame consists of columns made from IPE 220 and a cross-beam made from IPE 270. The frame has dimensions of 6 m in width and height. The columns are hinged, allowing deformation but rotation is fixed. The vertical load on both columns is identical.

Buckling analysis of the steel plane frame is performed using the beam finite element analysis software developed by the author. The computational model for beam finite elements is depicted in Fig. 2.

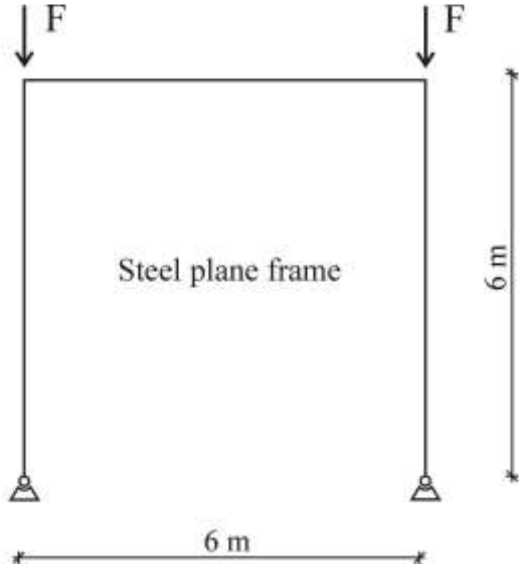

Fig. 2 Loads actions, boundary conditions, and member axes.

In the fundamental buckling mode, the frame bends uniformly, resembling a single half-sine wave, see Fig. 3. This mode represents the lowest eigenmode at which the frame can deform.

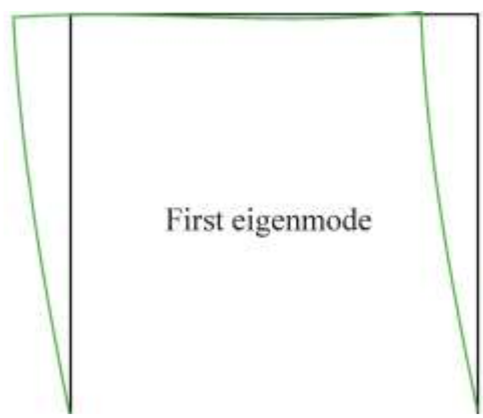

Fig. 3 First buckling mode.

The frame exhibits a more intricate deformation pattern as we progress to the second eigenmode. As we further explore higher-order eigenmodes, each additional mode introduces more sine waves into the deformation pattern, see Fig. 4, Fig. 5, Fig. 6, and Fig. 7. These additional sine waves correspond to extra cycles of bending or flexure along the length of the frame.

Two fundamental buckling modes can be distinguished based on observed behaviour. In the case of symmetric buckling, the structure does not exhibit any lateral deflection, see Fig. 4 and Fig. 6. However, in the unsymmetrical case, there is a noticeable lateral deflection that occurs from the structure's initial geometry, see Fig. 3, Fig. 5 and Fig. 7.

From a mathematical point of view, entropy is a specific additive functional applied to probability distributions:

$$H(Y) = -\sum P(y_i) \cdot \log_b(P(y_i)) . \qquad (1)$$

where $P(y_i)$ is the probability of discrete random variable Y, [12], [13]. Equation (1) forms the core of both thermodynamic entropy and Shannon's information entropy, differing only by a constant scaling factor applied to the equation, [14].

Can entropy be utilized to examine the characteristics of eigenmodes? Each eigenmode represents the shape of deformation in a buckled frame, independent of scale. The probabilities quantify the chance of a node or element undergoing a specific deformation. By normalizing deformations to a sum of one, each node's value corresponds to the probability of its particular deformation.

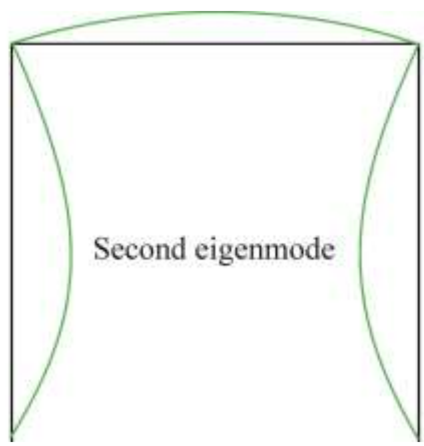

Fig. 4 Second buckling mode.

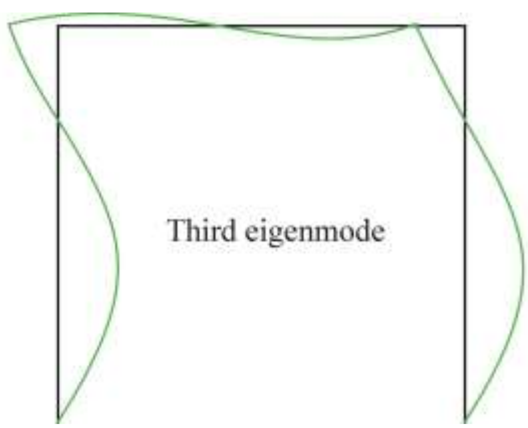

Fig. 5 Third buckling mode.

The number of eigenmodes plays a significant role in determining the number of waves and, subsequently, the impact on entropy values. As the number of eigenmodes increases, more distinct wave patterns emerge within the system. These additional waves introduce greater complexity and variation in the deformation distribution, influencing the overall entropy. Consequently, higher numbers of eigenmodes tend to result in lower entropy values. Considering the relationship between eigenmodes, wave patterns, and entropy, can provide valuable insights into the structural behaviour and understanding of the degree of freedom within the system.

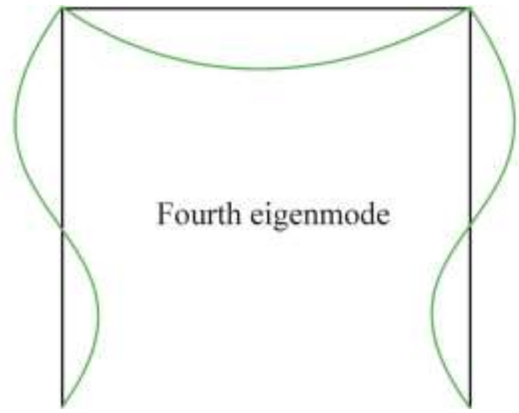

Fig. 6 Fourth buckling mode.

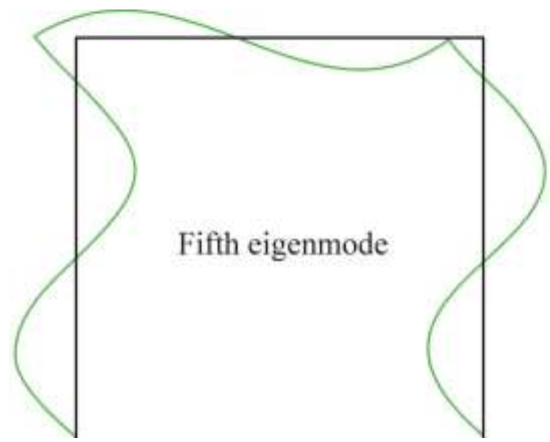

Fig. 7 Fifth buckling mode.

The principles of maximum entropy are applicable due to considering virtual displacements as probabilities. Uniform probability yields maximum uncertainty and, therefore, maximum entropy. Entropy, then, can only decrease from the value associated with uniform probability.

B. Computation of entropy from eigenmodes

The frame deformations are computed at 361 points, and the nodal displacements establish a discrete probability distribution for the entropy calculation. All beam elements are uniformly meshed with nodes. Fig. 8, Fig. 9, Fig. 10, Fig. 11 and Fig. 12 illustrate the transformation of the buckling mode into a discrete probability distribution at the nodes.

The entropy value increases as the discrete probability distribution approaches a uniform distribution. In Shannon theory, achieving an equal absolute deformation of 1/361 for each of the 361 nodes would result in the entropy reaching its maximum value of 5.889. Conversely, the entropy decreases as the deformations between nodes differ more significantly.

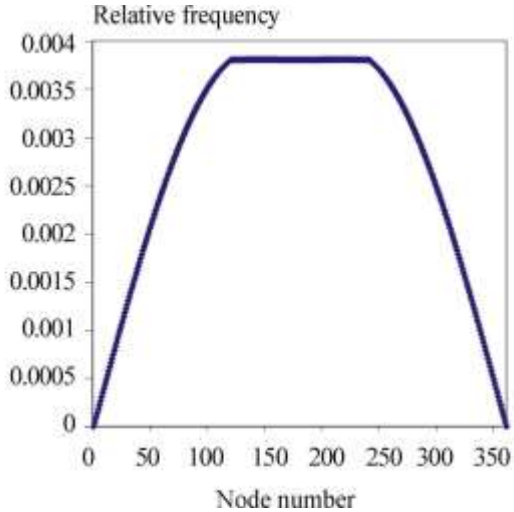

Fig. 8 Discrete probability distribution of first buckling mode.

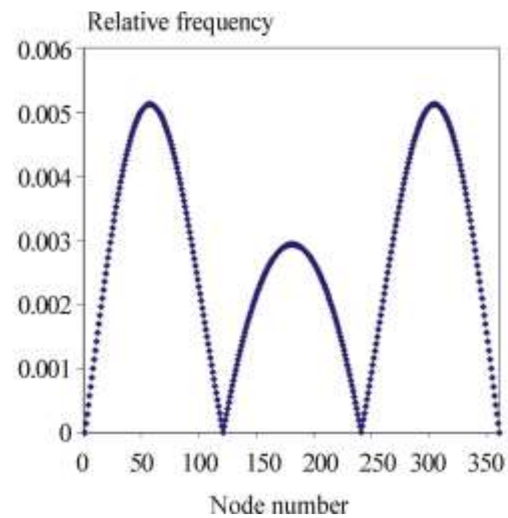

Fig. 9 Discrete probability distribution of first buckling mode.

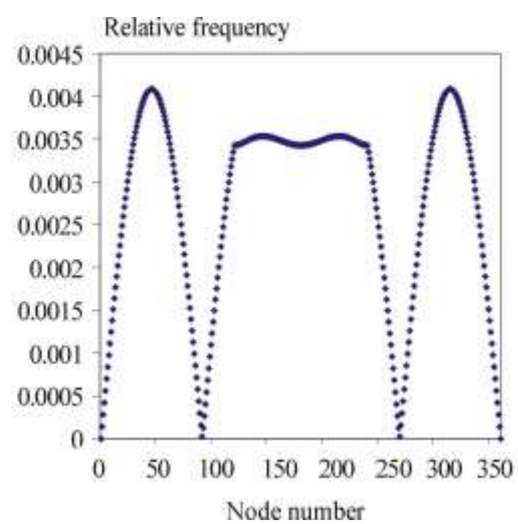

Fig. 10 Discrete probability distribution of first buckling mode.

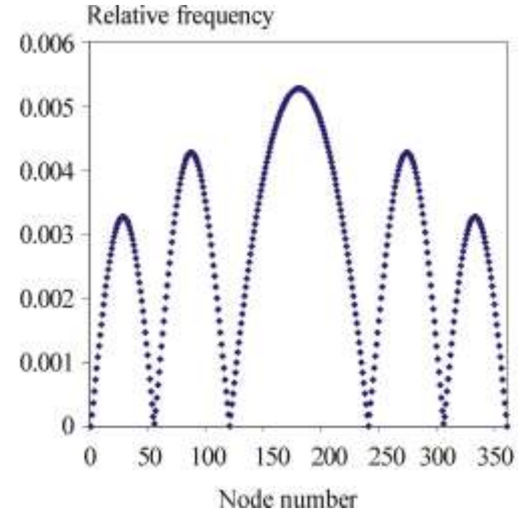

Fig. 11 Discrete probability distribution of first buckling mode.

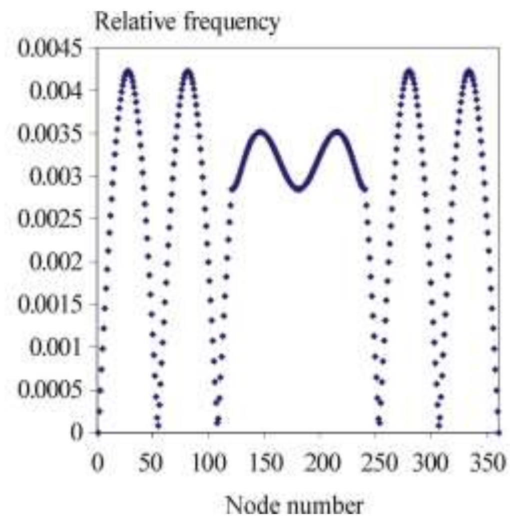

Fig. 12 Discrete probability distribution of first buckling mode.

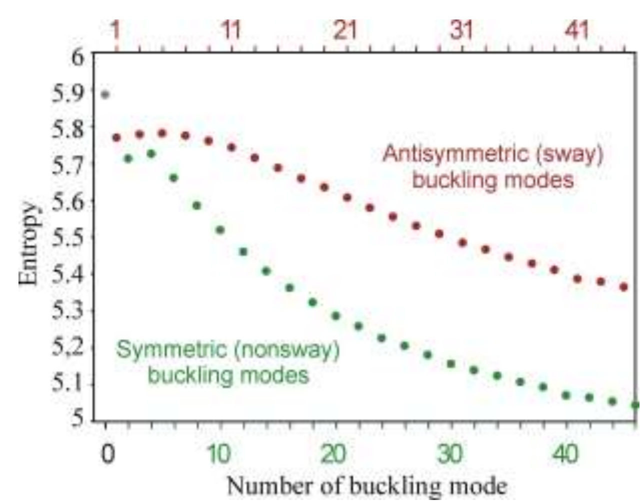

Fig. 13 Entropy for 47 buckling modes.

Fig. 13 depicts the evaluation of entropy for 47 buckling modes. Higher entropy values correspond to anti-symmetric (sway) buckling modes. Symmetric buckling modes exhibit lower entropy values compared to antisymmetric buckling

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modes, with entropy values of 5.769 for the first eigenmode, 5.778 for the third eigenmode, 5.782 for the fifth eigenmode, and 5.777 for the seventh eigenmode. The red data points with higher entropy may indicate a greater degree of freedom in the frame system and suggest using sway modes as more important initial imperfections.

III. Modelling of Initial Geometric Imperfections

Initial geometric imperfections can significantly influence the behaviour of elastic structural systems under compressive load. This involves introducing an equivalent imperfection in the form of an initial sway and individual bow imperfections for the members.

In the approach of most design standards, a method based on the general origin of inevitable assembly inaccuracies is preferred. Such forecasting of substitute global geometrical imperfection is derived from considering a steel frame with joints offset relative to the vertical lines originating at the support points, see Fig. 14. Specifically, Fig. 14 does not illustrate the inclusion of individual bow imperfections that are added separately.

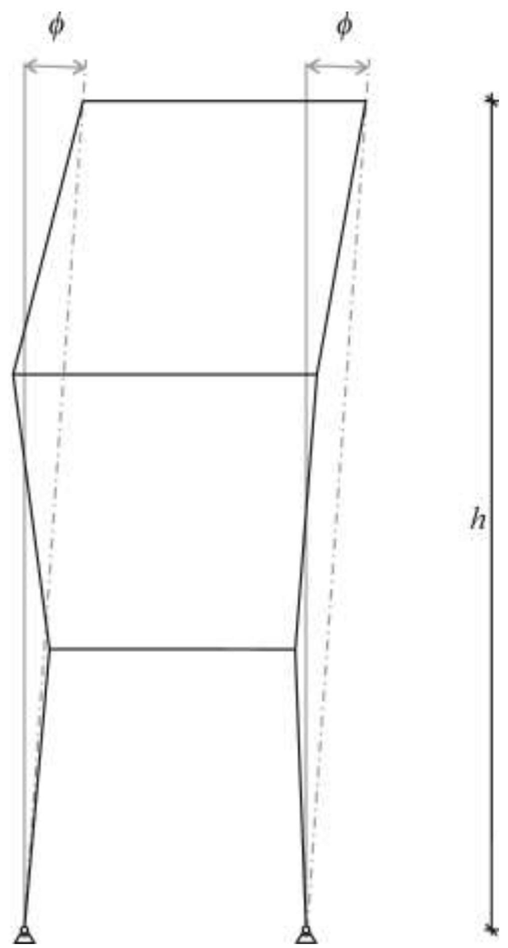

Fig. 14 The concept of initial sway imperfection.

Eurocode 3, [7], provides the calculation of global initial sway imperfections in the form of Equation (2).

$$\phi = \phi_0 \cdot \alpha_h \cdot \alpha_m \, . \qquad (2)$$

where $\phi_0 = 1/200$ is the basic value, $\alpha_h = 2/(h^{0.5})$ but $2/3 < \alpha_h < 1$, $\alpha_m = (0.5\cdot(1+1/m))^{0.5}$. The factor m represents the count of columns in a row, considering only those columns that bear a vertical load equal to or greater than 50 percent of the average value of the column in the considered vertical plane. The parameters α_h, α_m can be set differently, as illustrated, for example, in the approach α_h =1, $\alpha_m = m^{-0.455}$, [15], [16].

If standard approaches are too conservative, initial imperfections can be introduced using buckling modes. The sensitivity of the frame to sway imperfections depends on the shape of the first or several initial buckling modes. In the context of rectangular frames, the non-sway (symmetric) buckling mode often exhibits stable symmetric bifurcation, which is relatively insensitive to imperfections. However, in the case of the sway (asymmetric) buckling mode, certain rectangular frames have demonstrated sensitivity to imperfections, [9].

To simplify the definition and input of initial geometric imperfections for advanced analysis, [17], proposed a new procedure. This innovative approach involves defining initial imperfections as a linear combination of multiple buckling modes, where each mode is assigned a suitable amplitude. Unlike the conventional method that only considers the first buckling mode, this new procedure incorporates numerous eigenmodes. By adopting this approach, imperfections are introduced in virtually all members of the structure, thereby triggering the instability associated with the actual failure mode, [17], [18].

Until now, no study has been presented that analyzes the shape and size of the initial frame imperfection without using advanced geometric and material non-linear analysis. The influence of the initial imperfection has been analyzed to date solely through sensitivity analysis of load-carrying capacity, [19], [20]. A simple method to estimate the effect of this imperfection on the load-carrying capacity, based solely on the shape of the initial imperfection without analyzing the bearing capacity, is missing. If the shape is known, the use of tolerance limits would ensure that imperfections do not exceed defined limits with a predetermined probability.

This article introduces a methodology that utilizes entropy as a valuable tool for categorizing the buckling modes of steel plane frames. Each buckling mode is scaled in this approach based on the potential energy and entropy. The initial imperfection is then derived as a linear combination of the scaled buckling modes obtained through an elastic buckling analysis. By incorporating entropy as a quantification metric, this methodology provides a comprehensive understanding of the buckling behaviour and enables a more accurate representation of initial imperfections in steel plane frames.

A. The potential energy of buckling modes

The mechanical energy stored in a structure during buckling deformation is known as the potential energy of the buckling mode. When the structure fails, this energy is often released abruptly and explosively, leading to significant damage and injury.

Lagrange's variational principle is employed to analyse the buckling behaviour of steel frames. This principle considers the potential energy associated with the buckling mode, representing the elastic potential energy stored in the frame due to deformation under compressive loads. This potential energy is defined as a function of the elastic deformation of the steel plane frame and is typically expressed as follows:

$$\Pi_i = \int_0^l E \cdot I \cdot (w'')^2 dx \cdot \tag{3}$$

where E is Young's modulus, l is the beam element length, I is the second moment of area, and w represents the curve of the normalized elastic deformation resulting from buckling.

Fig. 15 shows the plot of the deformation due to buckling. Fig. 16 shows the plot of the bending moment due to buckling.

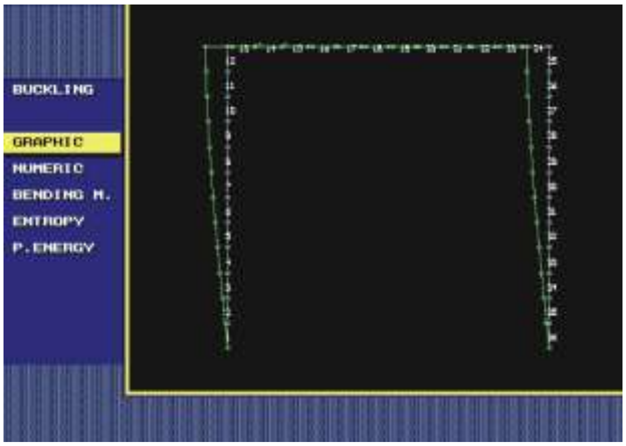

Fig. 15 The first buckling mode computed using FEM.

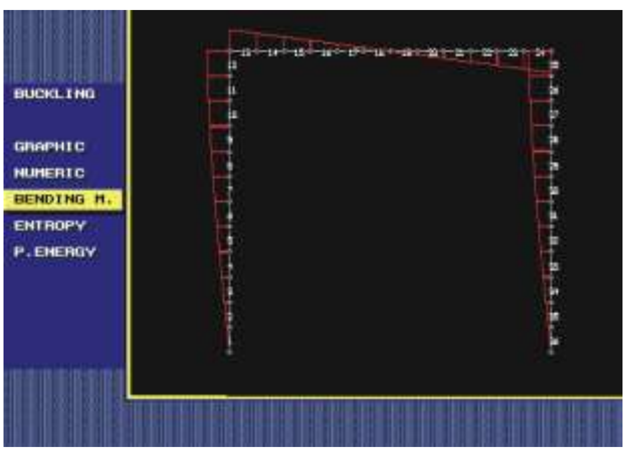

Fig. 16 Bending moment derived from first buckling mode.

In Fig. 15 and Fig. 16, we depict the deformation and bending moment caused by buckling without any scaling applied. It can be noted that the plot of the bending moment is the same as the plot of the second derivative of the normalized elastic deformation w in Eq. (3), see Fig. 16.

The potential energy of the whole steel plane frame is calculated for all finite elements as the sum of the elastic potential energy of all its elements.

B. *Entropy and potential energy of buckling modes*

In physics, the presence of an entropic force within a system arises as an emergent phenomenon driven by the system's statistical inclination to increase its entropy, [21]. This force materializes due to alterations in the entropy associated with the positions of particles, giving rise to various instances such as diffusion, gas pressure on a surface, osmosis, and even body elasticity, [22].

In the domain of macroscopic physics, the entropic force can be linked to the mechanical work contingent upon the system's entropy. These forces can be viewed as the collective outward influence exerted by a multitude of particles. At its core, the concept of entropic force establishes a connection between mechanical work, entropy, and temperature, [23].

$$F\Delta x = T\Delta S, \tag{4}$$

where F represents force, Δx denotes displacement, and T signifies temperature. The entropic force showcases its interplay with mechanical work while emphasizing the significance of entropy alterations. This concept has triggered intriguing discussions and has provided novel insights, [24], [25]. It can be noted that mechanical work is the foundation for numerous theories in structural mechanics. Despite the article, [23], already amassing 765 citations on the Web of Science and exploring mechanical work, its applications within structural mechanics remain noticeably absent.

In structural mechanics, mechanical work arises from the elastic deformation of a body. When a static force is exerted on a structure, it induces deformation, thereby altering the structure's shape. The work conducted by the applied force corresponds to the potential energy stored within the object due to its deformation.

Although the link is not direct, the left-hand side of Equation (4) can be interpreted as the potential energy encapsulated within the internal forces. On the right-hand side of Equation (4), the entropy can be understood as the entropy of the deformed body, calculated only for elements where the buckling-induced bending moment is non-zero. While deformation lacks an actual temperature, the pseudo-temperature can be perceived as an amplification factor when the potential energy and entropy are known.

To facilitate the comparative analysis of buckling modes, a methodology can be presented by introducing units of entropy and pseudo-temperature. It is crucial to note that the buckling mode does not possess an actual temperature. Consequently, the utilization of Equation (4) can be regarded as a heuristic application of a mathematical model wherein the potential energy is assigned an amplitude (pseudo-temperature) contingent upon the deformation shape.

Table 1 shows the calculated potential energy and entropy (using the natural logarithm). Table 2 presents the calculated pseudo-temperature T and scale for each buckling mode. The pseudo-temperature for each buckling mode is computed using Equation (4). Subsequently, the corresponding buckling mode undergoes a reduction in its normalized deformation scale so that its pseudo-temperature matches that of the first buckling mode.

TABLE 1. POTENTIAL ENERGY AND ENTROPY FOR BUCKLING MODES

Mode	Critical load [kN]	Potential energy [kJ]	Entropy
1	584.36	1.57	5.7686
2	4118.02	87.24	5.7144
3	5401.93	142.6	5.7781
4	13678.32	635.6	5.726
5	15556.63	1224.0	5.7819
6	29317.69	1970.6	5.6608
7	31515.44	4756.4	5.7774
8	51147.28	4280.4	5.5878
9	53515.29	12166.0	5.7634
10	79192.64	7675.7	5.5203
11	81657.36	24460.9	5.7435
12	113461.56	12220.8	5.4612

TABLE 2. SCALE FACTORS CALIBRATED FOR BUCKLING MODES

Buckling mode	Pseudo-temperature [K]	Scale factor [%]
1	0.2722	100
2	15.2667	13.3414
3	24.6794	10.4928
4	111.0024	4.9476
5	211.6951	3.5828
6	348.1133	2.7939
7	823.2769	1.8168
8	766.0260	1.8835
9	2110.9068	1.1346
10	1390.4498	1.3980
11	4258.8840	0.7988
12	2237.7500	1.1020

The last column in Table 2 provides the sensitivity scale of normalized buckling modes, which is useful for determining the initial geometric imperfection. The scale exhibits slight oscillations as the buckling mode increases. The trend of the scale is inversely correlated with that of the potential energy. The potential energy has the most substantial impact on the scale. Higher buckling modes exhibit greater potential energy, particularly for anti-symmetric buckling modes. As the number of modes increases, odd anti-symmetric modes have smaller scales compared to subsequent even modes. The initial frame imperfection created from the last column in Table 2 is shown in Fig. 17. The entropy of the initial imperfection has a value of 5.8167, which is the maximum value among all the entropies of individual buckling modes.

The decrease in scale can be interpreted as a "cooling" of the deformation associated with the corresponding buckling mode to match the pseudo-temperature of the first buckling mode. Higher pseudo-temperatures are linked to smaller scales. Although the differences in entropies among the buckling modes may not be significant, the odd (sway) buckling modes tend to exhibit higher entropy compared to the even (non-sway) buckling modes. The initial geometric imperfections are substantially impacted by the first few anti-symmetric buckling modes.

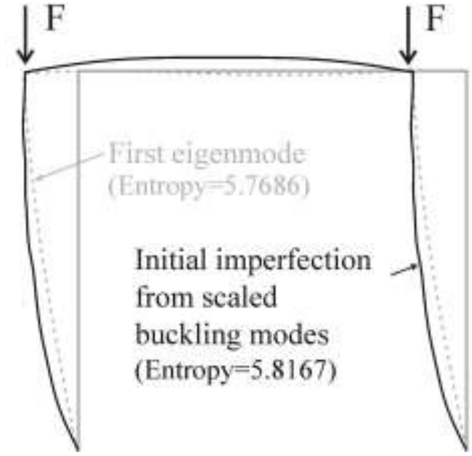

Fig. 17 Initial imperfections created from buckling modes.

The relationship between pseudo-temperature and the critical load is remarkable, see Fig. 18. The anti-symmetric (sway) buckling modes exhibit a weakly nonlinear behavior, while the symmetric (non-sway) buckling modes have an approximately linear behavior. One can question whether pseudo-temperature can have a similar significance as the critical load. The greater the pseudo-temperature, the greater the critical load. Both the critical load and the pseudo-temperature introduce energy into the system, which has the potential to sustain the deformation of the corresponding buckling mode.

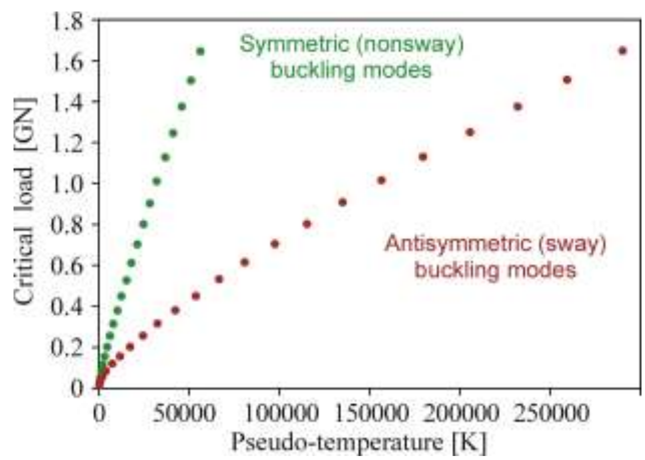

Fig. 18 Initial imperfections created from buckling modes.

This approach employs entropy as a new sensitivity measure for characterizing the shape of the initial imperfections and examining their influence on the frame's behaviour. It is worth noting that modifying the base of the logarithm in Equation (1) would alter the magnitudes of entropy and, consequently, the pseudo-temperature. However, despite such changes, the relative ratios in the Entropy column and the pseudo-temperature column would remain unaffected. Consequently, the scale factors presented in the last column of Table 2 would remain unchanged.

The accurate modelling of initial geometric imperfections is paramount when analysing the behaviour of steel plane frames.

The first buckling mode is commonly regarded as the most critical, thus serving as the foundation for introducing the initial imperfection. However, it is crucial to investigate the relative contribution of several important eigenmodes, starting with the first mode. Particularly in orthogonal frames, the non-sway (symmetric) buckling mode typically exhibits insensitivity to imperfections, whereas the sway (anti-symmetric) buckling mode demonstrates greater susceptibility, [9].

A comprehensive exploration of the effects of initial imperfections can be conducted using advanced techniques such as global sensitivity analysis methods tailored to load-carrying capacity as model output, failure probability, or the shape of the distribution, [26], [27], [28]. Alternatively, more sophisticated stochastic approaches involve modelling initial imperfections through the utilization of random variables or random fields, [29]. However, including correlated random inputs may also necessitate considering the impact of correlations among these variables, which is not typically accounted for in conventional sensitivity analysis methods, [30]. Investigating the entropy of buckling modes contributes to a deeper understanding of the scales associated with different types of imperfections, which are crucial factors in generating comprehensive imperfections that influence the overall behaviour of the frame.

IV. The Entropy in the Structural Mechanics

A. Elastic stability of structures and entropy

The presented article introduces a methodology that harnesses the power of entropy to quantify the buckling modes of steel plane frames. The first results obtained from the case study validate the effectiveness of this method.

Researching the entropy of buckling modes can enhance our understanding of the scales associated with sway (out-of-plumb) and bow imperfections, which are crucial components in generating global imperfections for the entire frame. It would be appropriate to investigate the presented ideas through further studies involving different types of frame constructions. In an alternate frame configuration with bracing stiffness presented in [10], the entropy of the first symmetric (non-sway) buckling mode can be lower than that of a specific anti-symmetric (sway) mode, contrary to the example presented in the article.

It can be expected that the methodology will work well for standard frames with standard loads. However, imperfections introduced on the unloaded bars or significant differences in stress distribution caused by the non-loading of certain bars, such as unloaded cantilevers or loading limited to the lower floor, can lead to buckling in the rest of the frame on higher floors. A simple example can be a frame supplemented with a cantilever that should not be included in the entropy calculation since its deformation modifies the entropy for certain buckling modes, without the cantilever resisting the compressive load or playing a role in introducing initial imperfections, see Fig. 19.

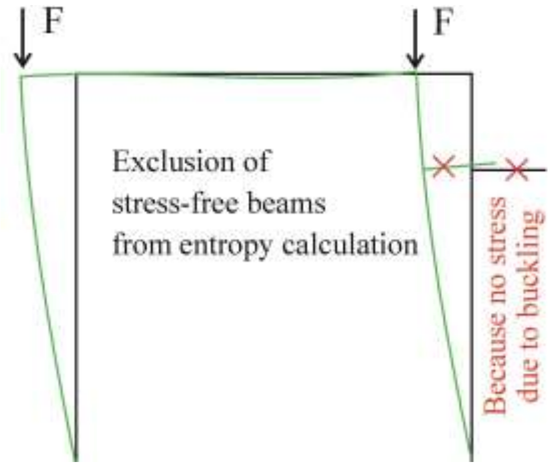

Fig. 19 Beam excluded from entropy calculation.

There will still be a lot of work to create rules for using entropy as a useful measure of the importance of initial imperfections. By introducing random imperfections, the correlation between the entropy of random realizations of initial imperfections and the structural resistance will be studied. The working hypothesis is that higher entropy leads to lower frame resistance, and it will need to be verified. Further case studies should include the analysis of different braced and unbraced, regular, and irregular frames with varying numbers of floors, stiffness, and geometry.

B. Lagrangian mechanics and entropy

Energy induces changes in the entropy of a system. Lagrange's principle of minimum potential energy strives to identify an equilibrium state with the least potential energy within the system. Potential energy can be perceived as a form of arrangement or structure within a system. This implies the existence of a relationship between entropy and potential energy, given that both concepts are rooted in the arrangement and alterations of this arrangement within the system.

Fundamental research poses unanswered questions. The potential energy forms the foundation of Lagrange's principle of minimum potential energy. Does Verlinde's proposition, [22], hold, and could there exist a more general connection between potential energy and entropy? Can the internal forces within beams be perceived as entropic forces linked to the deformation of bodies? This intriguingly raises the question of whether it would be possible to reformulate Lagrange's principle by incorporating entropy within the deformation variation principle. However, it is important to note that these concepts remain distinct and cannot be directly linked without a thorough examination of the specific context and conditions involved.

There exists a notable connection between Lagrange's principle of minimum potential energy and entropy, as both seek the extrema of certain variables. While Lagrange's principle seeks the deformation that minimizes the potential energy of a structure, the second law of thermodynamics dictates that entropy in a closed system never decreases but rather increases. If Lagrange aims to minimize the total potential energy, could the entropy principle, in turn, seek to maximize entropy?

V. Conclusion

This study presented a novel approach for modelling initial geometric imperfections in steel plane frames. Initial imperfections were introduced by analysing buckling modes, and normalised deformations, and their scales were assessed using Shannon entropy and potential energy analysis. The case study demonstrated a decreasing scale of the buckling modes, providing novel insights into the behaviour of steel frames.

The initial imperfection shown in Fig. 17 has the highest entropy among all buckling modes. If entropy is considered one of the indicators of the danger of the initial imperfection, the introduced algorithm can be an effective tool for practically introducing imperfections into all structural elements and triggering instability associated with the actual failure mode. The established algorithm produces an initial imperfection with large entropy, but future research may also focus on imperfections with an average, quantile, or optimal entropy.

Compared to methodologies employed by other researchers, the proposed methodology presented here does not require statistical analysis or output from a finite element model. Initial geometric imperfections are solely generated using buckling modes. The results of the case study demonstrated the dominant influence of the first buckling mode, similar to findings by other researchers, [17], [18].

The analysis of buckling modes revealed that anti-symmetric buckling modes exhibited higher entropy than symmetric buckling modes. This entropy-based analysis proved valuable, especially when the critical buckling loads of non-sway and sway buckling modes were closely aligned or overlapped. The analysis of entropy based on the deformation of buckling modes offered a new perspective and a deeper understanding of their complexity in steel frames.

The study highlighted that entropy provides valuable information about buckling modes obtained from elastic buckling analysis. By examining the probability distribution associated with the deformations of buckling modes, entropy could be calculated as a measure of the degree of disorder or complexity of buckling in steel plane frames. Moreover, the sway (anti-symmetric) buckling modes were found to have greater entropy than the non-sway (symmetric) buckling modes.

Initial imperfections were introduced by scaling each buckling mode using potential energy and entropy. Although potential energy exerted a dominant influence on the scale, entropy played a crucial role in distinguishing detailed differences in the shape of individual buckling modes. This distinction was particularly useful when two buckling modes were closely aligned or coincided. In such cases, a perfectly straight frame would tend to buckle into a more probable state with higher entropy rather than solely aiming for a state of lower potential energy. Consequently, higher entropy corresponded to a higher scale of the buckling mode in the sum for the initial geometric imperfection.

The study proposed a novel methodology that utilized the entropy of frame-normalized deformation as additional information for computing the scales of buckling modes. This approach relied on the heuristic implication of the relationship between entropy and potential energy, which has broad applicability across scientific and technological domains.

The case study results demonstrated a clear relationship between critical force and pseudo-temperature. Further investigation can delve into more detailed connections between critical force, pseudo-temperature, entropy, and mechanical work. By studying these relationships in greater depth, a deeper understanding of the generation of initial geometric imperfections can be achieved.

Overall, the presented methodology and findings contribute to advancing the understanding and modelling of initial geometric imperfections in steel plane frames. They provide a foundation for future research and application of entropy-based analysis in structural engineering and related fields. The introduced concepts show potential for further advancements and can be extended to analyze and optimize other structural systems influenced by stability.

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Conflict of Interest

The author has no conflict of interest to declare.