Development of methods and computational algorithms parallelepiped in the presence of temperature and heat exchange

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Abstract—The article describes computational algorithms for estimating the law of distribution of body temperature in the form of a rectangular parallelepiped. The case is studied when a conditioned temperature is maintained on one of the boundaries of a rectangular parallelepiped, and heat exchange with the environment occurs on the opposite side. In addition, there are cases when other faces of the parallelepiped are thermally insulated or are under the influence of the environment. A polynomial is chosen as the approximating function. In accordance with the proposed layout, a function is formed that considers temperature, heat exchange with the environment, and insulation of the faces of a rectangular parallelepiped. The temperatures at the nodal points are determined by minimizing the function. Further, the temperature distribution law is determined according to the proposed approximating polynomial. The estimation of temperature distribution law is calculated for different amounts of partitioning into elements of a rectangular parallelepiped.

Keywords—Approximation, heat transfer, isolation, solid, rectangular parallelepiped.

I. INTRODUCTION

Much attention is paid to the problems of thermal conductivity in the Republic of Kazakhstan and abroad.

The paper, [1], presents the joint tasks of applying the finite element method to determine the thermomechanical data of various solids.

In, [2], a method and a computational algorithm for finding the temperature field along the length of a rod of narrow length and variable cross-section are proposed. There is also an analytical solution to this problem.

The paper, [3], presents an energy method for finding the law of temperature distribution, three forming deformations, and stresses in a rod of an unstable cross-section, provided that both ends are firmly fixed.

In, [4], a stationary solution of thermal conductivity problems is obtained for a rectangle with specified initial zero temperatures. The method of time separation is proposed, and its effectiveness in solving this problem is shown.

In, [5], exact nonstationary solutions to the topic of thermal conductivity in two-dimensional rectangles heated at the boundary are investigated. A summation method is proposed that guarantees the best convergence at and near the hot boundary.

The paper, [6], shows the temperature distribution in a rectangular parallelepiped is considered using the finite difference method. Joint formulations for three-dimensional and non-stationary heating of a finite continuous rectangular parallelepiped under the influence of a free-sized heat source and an arbitrary initial temperature distribution are presented when convective boundary conditions suitable from time to time are established on six straight surfaces.

An analytical solution for determining the temperature distribution law in a rectangular parallelepiped using the function I was acquired in, [7].

The study of thermal conductivity in a rectangular parallelepiped is devoted to the work, [8]. It is noted that the thermal conductivity in solids of a free geometric shape, such as a rectangular parallelepiped, is of great importance because such shapes are very common in practice. In this article, an analytical conclusion is acquired for a simple particular problem.

The use of the Jacobi polynomial and the multidimensional Aleph function for thermal conductivity in an inhomogeneous moving rectangular parallelepiped is considered in, [9].

In, [10], the case is considered when the heat flow enters one of the faces of a rectangular parallelepiped, and heat exchange with the environment occurs on the opposite face. At the same time, options are being considered when the remaining faces of a rectangular parallelepiped are thermally insulated or vice versa. The laws of temperature distribution are obtained when a rectangular parallelepiped is divided into a different number of elements. It is shown that acceptable reliability is achieved already by dividing the edges of a rectangular parallelepiped into three or 4 parts.

II. THE AIM AND OBJECTIVES OF THE STUDY

Consider a solid body in the form of a rectangular parallelepiped in Figure 1. The origin of the coordinates is located in the lower-left corner of a rectangular parallelepiped (node 0), as shown in the figure. The vertices are numbered starting from node 0. The dimensions of a rectangular parallelepiped along the x, y, and z axes are considered equal to a, b, and c, respectively. Convective heat exchange occurs on the face (0, 1, 2, 3), and a constant temperature T* is maintained on the face (4, 5, 6, 7).

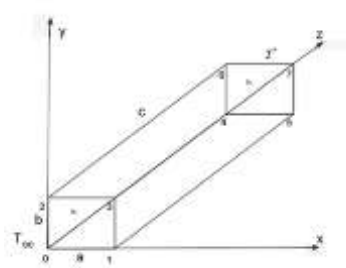

Figure 1. A solid body in the form of a rectangular parallelepiped

At the same time, it is considered that the faces (0, 1, 4, 5), (1, 3, 7, 5), (2, 3, 6, 7), (0, 1, 4, 6) are thermally insulated.

The equation of thermal conductivity for the body in question has the form:

$$K_{xx}\left(\frac{\partial^2 T}{\partial x^2}\right)+K_{yy}\left(\frac{\partial^2 T}{\partial y^2}\right)+K_{zz}\left(\frac{\partial^2 T}{\partial z^2}\right)=0, \quad (1)$$

$$K_{xx}\left(\frac{\partial T}{\partial x}\right)+K_{yy}\left(\frac{\partial T}{\partial y}\right)+K_{zz}\left(\frac{\partial T}{\partial z}\right)\Bigg|_{S_2}+h(\mathrm{T}-\mathrm{T}_0)=0, \quad (2)$$

$$T\big|_{S_1}=T^* \quad (3)$$

where T – temperature, ${}^0C$;

K_{xx}, K_{yy}, K_{zz} – thermal conductivity coefficients along the x, y and z axes, $\frac{kWt}{m\,{}^0C}$;

h – heat transfer coefficient, $\frac{kWt}{m^2\,{}^0C}$;

S_1 – surface, in m^2, where the temperature is set T^*;

S_2 – surface, in m^2 which the heat exchange takes place;

T_{amb} – ambient temperature, ${}^0C$.

In relation Equations (2) and (3) are boundary conditions, and Equation (2) characterizes convective heat transfer on the face (0, 1, 2, 3) of a rectangular parallelepiped (area S_1, and Equation (3)) is the temperature set on the edge (4, 5, 6, 7) – (area S_2).

The problem is to determine the law of temperature distribution in a solid of the shape of a rectangular parallelepiped.

III. RESEARCH METHODOLOGY

The solution of the task consists of the following stages:

1. Determination of the function of the final thermal energy characterizing the process under study, taking into account the effect of temperature and the presence of heat exchange on the opposite sides of a rectangular parallelepiped. At the same time, other faces can be heat-insulated and non-heat-insulated.
2. Development of a method for sampling a body in the form of a rectangular parallelepiped.
3. Development of a method for constructing an approximating temperature polynomial from three variables.
4. The concept of a perfect thermal energy functional using constructed approximating polynomials from three unstable ones and taking into account discretization.

Development of a method for the theory of enabling systems of linear algebraic equations based on the minimization of the constructed functional, taking into account natural boundary conditions.

Development of a method for deriving acquired solving systems of linear equations.

Development of Python code for estimating the body temperature distribution law in the form of a rectangular parallelepiped.

According to the variational principle, the solution of the problem under consideration is equivalent to minimizing the functional, [1]:

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$$J = \int_V \frac{1}{2}[\mathrm{K}\,xx(\frac{dT}{dx})^2 + \mathrm{Kyy}(\frac{dT}{dy})^2 + \mathrm{Kzz}(\frac{dT}{dz})^2]dv +$$
$$\int_{S_2} \frac{h}{2}(\mathrm{T}-\mathrm{T}_{amb})^2 dS = J_1 + J_2 + J_3 + J_4 \quad (4)$$

For a rectangular parallelepiped, formula (4) has the form:

$$J = \frac{1}{2}\int_0^a\int_0^b\int_0^c[\mathrm{K}\,xx(\frac{dT}{dx})^2 + \mathrm{Kyy}(\frac{dT}{dy})^2 + \mathrm{Kzz}(\frac{dT}{dz})^2] +$$

$$\frac{h}{2}\int_0^a\int_0^b(\mathrm{T}-\mathrm{T}_{amb})^2\, dxdy\Big|_{z=0} = J_1 + J_2 + J_3 + J_4 \qquad (5)$$

When the side faces of a rectangular parallelepiped do not have thermal insulation, the following terms are added to the functional J:

$$J_5 = \frac{h}{2}\int_0^a\int_0^c(\mathrm{T}-\mathrm{T}_{at})^2 dydz\Big|_{y=0},$$
$$J_6 = \frac{h}{2}\int_0^a\int_0^c(\mathrm{T}-\mathrm{T}_{at})^2 dxdz\Big|_{y=b}, \qquad (6)$$
$$J_7 = \frac{h}{2}\int_0^a\int_0^c(\mathrm{T}-\mathrm{T}_{at})^2 dydz\Big|_{x=a},$$
$$J_8 = \frac{h}{2}\int_0^a\int_0^c(\mathrm{T}-\mathrm{T}_{at})^2 dydz\Big|_{x=0},$$

where J_5, J_6, J_7, J_8 – characterize the heat exchange on the faces (0, 1, 4, 5), (2, 3, 6, 7), (0, 2, 4, 6), (1, 3, 5, 7) a rectangular parallelepiped, respectively.
The heat transfer accounting code is shown in Figure 2 TL – the temperature of the parallelepiped on the left side, JL - the functions relating to the heat transfer on the left edge of the parallelepiped.

```
JL=sympy.integrate(h*(TL-Toc)**2/2,  (y,0,b))
JL=sympy.integrate(JL, (z,0,c))
```

Figure 2. Accounting for heat transfer of the left side

In Figure 2 the program code for calculating the function corresponding to the heat exchange the second integral of formula (4) on the left side of a rectangular parallelepiped is indicated by JL.

In this case, the total functional is $J = \sum_{i=1}^{8} J_i$.

To minimize the functional J, the temperature T (x, y, z) is approximated by a polynomial, [4]:

$$T(\mathrm{x,y,z}) = \varphi_0(\mathrm{x,y,z})*\mathrm{T}_0 + \varphi_1(\mathrm{x,y,z})*\mathrm{T}_1 +$$
$$\varphi_2(\mathrm{x,y,z})*\mathrm{T}_2 + \varphi_3(\mathrm{x,y,z})*\mathrm{T}_3 + \varphi_4(\mathrm{x,y,z})* \qquad (7)$$
$$\mathrm{T}_4 + \varphi_5(\mathrm{x,y,z})*\mathrm{T}_5 + \varphi_6(\mathrm{x,y,z})*\mathrm{T}_6 + \varphi_7(\mathrm{x,y,z})*\mathrm{T}_7$$

Where,

$$\varphi_0(\mathrm{x,y,z}) = 1 - \frac{z}{c} - \frac{y}{b} + \frac{yz}{bc} - \frac{x}{a} + \frac{xz}{ac} + \frac{xy}{ab} - \frac{xyz}{abc};$$
$$\varphi_1(\mathrm{x,y,z}) = \frac{x}{a} - \frac{xz}{ac} - \frac{xy}{ab} + \frac{xyz}{abc};$$
$$\varphi_2(\mathrm{x,y,z}) = \frac{y}{b} - \frac{yz}{bc} - \frac{xy}{ab} + \frac{xyz}{abc}; \qquad (8)$$
$$\varphi_3(\mathrm{x,y,z}) = \frac{xy}{ab} - \frac{xyz}{abc};$$
$$\varphi_6(\mathrm{x,y,z}) = \frac{yz}{bc} - \frac{xyz}{bc^2};$$
$$\varphi_5(\mathrm{x,y,z}) = \frac{xz}{ac} - \frac{xyz}{abc};$$
$$\varphi_6(\mathrm{x,y,z}) = \frac{yz}{bc} - \frac{xyz}{bc^2};$$
$$\varphi_7(\mathrm{x,y,z}) = \frac{xyz}{bc^2};$$

The program code for obtaining an approximating polynomial is shown in Figure 3.

```
A=Matrix([
    [0],
    [0],
    [0],
    [0],
    [0],
    [0],
    [0],
    [0],
    ])
A[ 0 ]= 1 - z/c - y/b + y*z/(b*c) - x/a + x*z/(a*c) + x*y/(a*b) -
    x*y*z/(a*b*c)
A[ 1 ]= x/a - x*z/(a*c) - x*y/(a*b) + x*y*z/(a*b*c)
A[ 2 ]= y/b - y*z/(b*c) - x*y/(a*b) + x*y*z/(a*b*c)
A[ 3 ]= x*y/(a*b) - x*y*z/(a*b*c)
A[ 4 ]= z/c - y*z/(b*c) - x*z/(a*c) + x*y*z/(a*b*c)
A[ 5 ]= x*z/(a*c) - x*y*z/(a*b*c)
A[ 6 ]= y*z/(b*c) - x*y*z/(b*c**2)
A[ 7 ]= x*y*z/(b*c**2)
T=A[0]*T0+A[1]*T1+A[2]*T2+A[3]*T3+A[4]*T4+A[5]*T5+A[6]*T6+A[7]*T7
```

Figure 3. Temperature approximation

Figure 3 shows the temperature approximation code (7) by the nodal points of a rectangular parallelepiped.
Here A[i] = $\varphi_i(\mathrm{x,y,z})$

Differentiating (8) by the variables x, y and z we get:

$$\frac{\partial T}{\partial x} = \sum_{i=1}^{7}\frac{\partial\varphi_i}{\partial x}T_i;$$
$$\frac{\partial T}{\partial y} = \sum_{i=1}^{7}\frac{\partial\varphi_i}{\partial y}T_i; \qquad (9)$$
$$\frac{\partial T}{\partial z} = \sum_{i=1}^{7}\frac{\partial\varphi_i}{\partial z}T_i;$$

Substituting the value of T from (7) and the value $(\frac{dT}{dx},\frac{dT}{dy},\frac{dT}{dz})$ from (9) we calculate the functional (5). It is calculated using the sympy module of Python.
The code for calculating the general function is shown in Figure 4:

```
dtx=sympy.diff(T,x)
dty=sympy.diff(T,y)
dtz=sympy.diff(T,z)

ux=sympy.integrate(0.5*Kxx*dtx**2, (x,0,a))
ux=sympy.integrate(ux, (y,0,b))
ux=sympy.integrate(ux, (z,0,c))
print("JKx=",ux,"\n")

uy=sympy.integrate(0.5*Kxx*dty**2, (y,0,b))
uy=sympy.integrate(uy, (x,0,a))
uy=sympy.integrate(uy, (z,0,c))
print("JKy=",uy,"\n")

uz=sympy.integrate(0.5*Kxx*dtz**2, (z,0,c))
uz=sympy.integrate(uz, (x,0,a))
uz=sympy.integrate(uz, (y,0,b))
print("JKz=",uz,"\n")
JXYZ=ux+uy+uz
```

Figure 4. General functional calculation code

Figure 4 shows the program code for calculating the first integral of (5).
To minimize the functional J, we differentiate it by variables $\overline{T_0, T_7}$ and we equate it to zero. The differentiation of the function is shown in Figure 7:

```
ID = [1] * n
for i in range(n):
    ID[i]=sympy.diff(J,T[i])
```

Figure 5. The differentiation code of the general functional

Figure 5 shows the differentiation code of the general functionality. Equating the acquired formulations to zero, we acquire a system of linear equations relative to the temperature at the nodal points of a rectangular parallelepiped.
A system of linear equations is solved with respect to variables $\overline{T_0, T_7}$. At the same time, if the temperature is set on the face (4, 5, 6, 7) $\left(T_4 = T_4^*, T_5 = T_5^*, T_6 = T_6^*, T_7 = T_7^*,\right)$, subsequently, we obtain a system of linear equations concerning variables $T_0 - T_3$.
The solution of the resulting system of equations makes it possible to determine the temperature values at the nodal points of a rectangular parallelepiped. Substituting these values in (7), we acquire the law of temperature distribution in a rectangular parallelepiped.
A general flowchart for determining the law of temperature distribution in the body of a rectangular parallelepiped is shown in Figure 6.

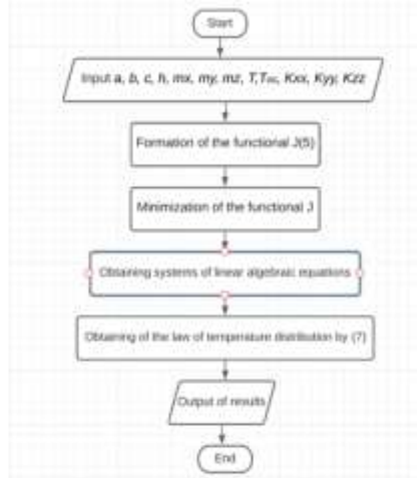

Figure 6. Block diagram for determining the temperature distribution law

To solve this problem, initial data are first entered and a function is compiled relating to the law of conservation of energy Figure 6. Subsequently, the concept of linear equations is compiled by minimizing the acquired function, the solution of which gives the meaning of the temperature at the nodal points of a rectangular parallelepiped. The concept of these temperature values in (7) allows us to determine the law of temperature distribution in the body in the form of a rectangular parallelepiped.
All calculations were obtained using a program developed in the Python programming language.

IV. Results

For the practical implementation of the proposed approach, a Python program was developed. As an example, a cube was selected in Figure 7 with the following initial data:

a = 0.06m, b =0.06m, c =0.06m, $K_{xx} = 75000 \frac{Wt}{m^0C}$,

$T_{amb} = 40\,^0C$, *T=300* ^0C, *h=100000* $\frac{kWt}{m^{2\,0}C}$.

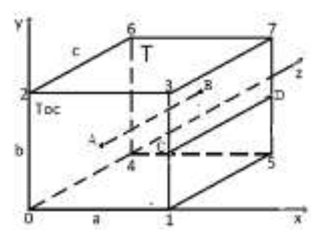

Figure 7. A solid body in the shape of a cube, consisting of a single element

If the number of partitions of a rectangular parallelepiped into elements along the x, y, and z axes is denoted as *mx, my и mz*, accordingly, then for the cube we have *mx=my=mz=m.*
Consider the partition of a cube on each face into m=3 elements in Figure 9. Let's introduce arrays of temperatures for this case *T1[0,8], T2[0,27], and T3[0,63]* corresponding to the nodal points of the cube. The results of calculating these

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temperature values for the thermally insulated case according to the proposed method turned out to be equal.
by m = 1 Figure 7:
T1[0,3] = 274.54, T1[4,7] = 300;
1) by m = 2 (Figure 8):
T2[0,8] = 274.54, T2[9,17] = 287.27, T2[18,27] =300;

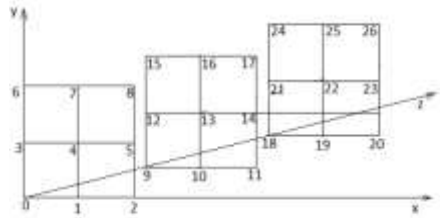


Figure 8. A solid body in the shape of a cube when divided into 8 elements

2) by m = 3 (Figure 9):
T3[0;15] = 274.54, T3[16;31] = 283.03, T3[32;47] = 291.51, T4[48,63] = 300;

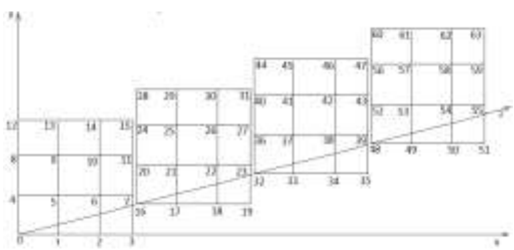


Figure 9. A solid body in the shape of a cube when divided into 27 elements

The temperature distribution laws for the thermally insulated case for segments (0, 4), (A, B), and (C, D) Figure 7 at m = 1, m = 2, and m = 3 turned out to be the same Figure 10:

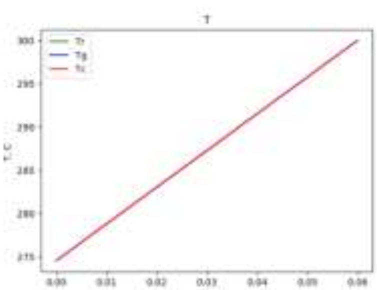


Figure 10. Temperature distribution law for the thermally insulated case at m = 1, m = 2, m = 3

In Figure 10 through Tr, Tg, and Tc temperatures are indicated by segments (0,4), (C, D), (A, B), accordingly.
The law of temperature distribution on the face (1, 3, 5, 7) of the cube for the thermally insulated case is shown in Figure 11.

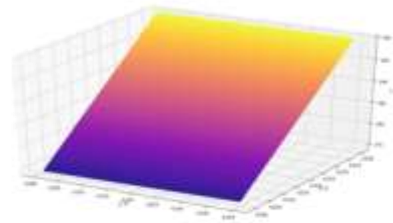
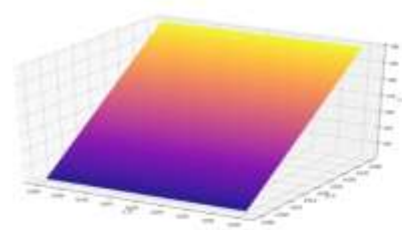


Figure 11. The law of temperature distribution on the face (1, 3, 5, 7) of the cube for the thermally insulated case.

Figure 11 shows that as the z value increases on the face (1, 3, 5, 7), the temperature increases rectilinearly from 274.54 до 300. The same result is obtained for all faces and sections of the cube parallel to the z-axis.
We now investigate the temperature distribution law for the non-insulated case.
The temperatures for the edges of the cube passing through the nodes were calculated:
- (0, 4) by m = 1. Here the temperature in the nodes is T1= [231.89, 300];
- (0, 9, 18) by m = 2. Here the temperature in the nodes is T2= [228.17, 254.79, 300];
- (0, 16, 32, 48) by m = 3. Here the temperature in the nodes is T3= [227.32, 243.74, 265.39,300].

The law of temperature distribution on the face (1, 3, 5, 7) of the cube for the non-insulated case at m = 1 is shown in Figure 12.



In Figure 12 the law of distribution of non-temperature on the face (1, 3, 5, 7) of the cube at m = 1 for the non-insulated case.

The law of temperature distribution on the face (1, 3, 5, 7) of the cube for the non-insulated case at m = 2 is shown in Figure 13.

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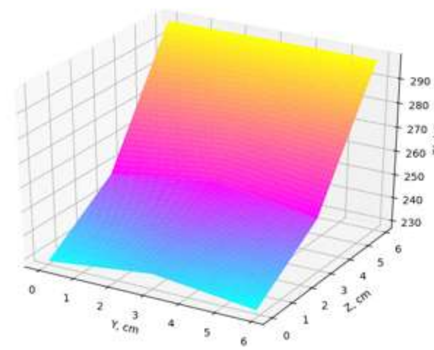

Figure 13. The law of distribution of non-temperature on the face (1, 3, 5, 7) of the cube at m = 2 for the non-insulated case.

The law of temperature distribution on the edge (1, 7, 19, 25) is in (Figure 8) and the cube for the non-insulated case at m = 2 is shown in (Figure 14).

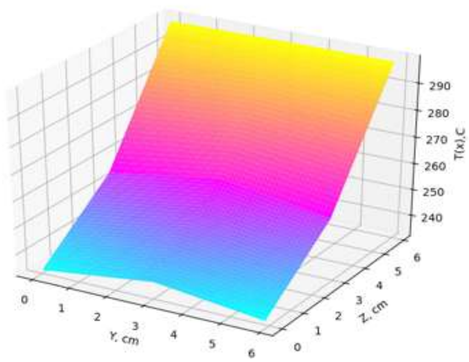

Figure 14. The law of distribution is not the temperature on the edge (1, 7, 19, 25) in Figure 8 a cube at m = 2 for the non-insulated case.

The laws of temperature distribution for the non-insulated case for segments (0, 4), (A, B), and (C, D) in (Figure 7) at m = 3 are shown in Figure 15.

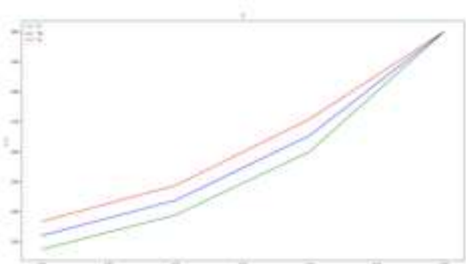

Figure 15. The temperature distribution law for the non-insulated case at m = 3.

Figure 15 shows that the temperature at the middle of the cube (Tc) is greater than the temperature at the middle of the face (Tg) and its turn (Tg) is greater than (Tr).
The law of temperature distribution in the cube for the edge (0, 4) in Figure 7 when partitioning m = 1, m = 2 и m = 3 in the non-insulated case is shown in Figure 16.

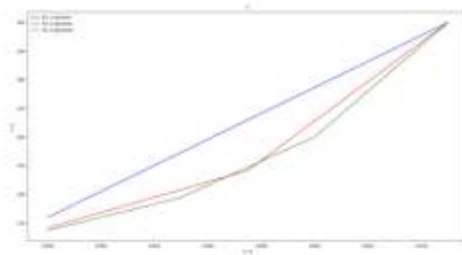

Figure 16. Temperature distribution laws for the edge (0, 4) of the cube in Figure 7 for the non-insulated case when m = 1, m = 2, m = 3.

Figure 16 shows that the temperature distribution law for the non-insulated case has a nonlinear character. Let's determine the maximum relative error between partitions 1 and 2. On the z axis when m = 1 ($T^1 = 267.5$) and by m = 2 ($T^2 = 248.3$).

$$\frac{T^1 - T^2}{T^2} 100\% = 7.7\%$$

Determine the maximum temperature deviation between partitions 2 and 3.
Determine the maximum relative error between partitions 2 and 3. Along the z axis at m = 2 (T^2=265) and by m = 3 (T^3 = 260).

$$\frac{T^2 - T^3}{T^3} 100\% = 2.8\%$$

V. Conclusion

According to the variational approach, a general function is obtained for the body of the shape of a rectangular parallelepiped under the influence of temperature on a certain face and heat exchange or thermal insulation of other faces for a given sampling number. By minimizing the acquired function for the temperatures of the main points of a rectangular parallelepiped, the concept of linear equations is determined. The solution of this system made it possible to acquire the temperature at the nodal points of a rectangular parallelepiped, and by substituting these values into the formula proposed by the polynomial approximation method, revealed the law of temperature distribution according to the shape of the body of a rectangular parallelepiped.

The actual execution of the investigated approach is performed on a clear example, when a continuous temperature is maintained on one of the faces of a rectangular parallelepiped, and heat exchange with the environment goes out on the opposite side. And the other faces are not represented by heat-insulated or vice versa. At the same time, the law of temperature distribution in different sections of the cube was investigated, and it was shown that the temperature of the section passing through the center of the cube is greater than that of other sections. the cases of splitting a rectangular parallelepiped along the z-axis into 1,2 and 3 elements were considered. It is shown that when divided into three elements, the relative error in determining the temperature does not exceed 2.8%.

References

[1] J. L. Segerlind, “Applied finite element analysis”, New York-London-Sydney-Toronto, Jonh Wiley and Song, 1976.

[2] A. Kudaykulov, A.A. Tashev, A. Askarova, “Computational algorithm and the method of determining the temperature field along the length of the rod of variable cross section”, Open Engineering, vol.8, 2018, pp. 170-175. J. L. Segerlind, “Applied finite element analysis”, New York-London-Sydney-Toronto, Jonh Wiley and Song, 1976.

[3] Arshidinova M, Begaliyeva K, Kudaykulov A, Tashev A, “Numerical Modeling Of Nonlinear Thermomechanical Processes in a rod of variable cross section in the presence of heat flow”, International Conference on Information Science and Control Engineering,2018, pp. 351-354.

[4] J. V. Beck, “Verification solution for partial heating of rectangular solids”, International Journal of Heat and Mass Transfer”, vol. 47, 2004, pp. 4243–4255.

[5] J. V. Beck, K. D. Cole, “Improving convergence of summations in heat conduction, International Journal of Heat and Mass Transfer”, vol. 50, 2007, pp. 257–268.

[6] Y. O. Nurettin, «General solution to a class of unsteady heat conduction problems in a rectangular parallelepiped», International Journal of Heat and Mass, vol. 12, 1969, pp 393-411.

[7] J. W. Vergnaud, J. Bouzon, «Heat conduction n a Rectangular parallelepiped», Cure pf thermosetting Resins, pp 89-93.

[8] D. Kumar, F. Y. Ayant, C. Cesarano, «Analytical Solutions of Temperature Distribution in a Rectangular Parallelepiped», Axioms, vol. 11, 2022, pp 488.

[9] D. Kumar, F. Y. Ayant, «Application of Jacobi Polynomial and Multivariable Aleph-Fuctional in heat conduction in Non-Homogeneous moving rectangular parallelepiped» Kragujevac Journal of Mathematics, vol 45, 2021, pp439-448.

[10] A. A. Tashev, A. K. Kudaykulov, R. K. Kazykhan, R. B. Aitbayeva, M. Arshidinova, M. M. Kunelbayev, D. A. Kuanysh, “Variational approach for estimating the temperature distribution in the body of a rectangular parallelepiped shape”, Vol. 16, pp. 10, (2022).

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

-Kazykhan Rysgul carried out the discretization of a rectangular parallelepiped along the x, y, and z axes and the construction of approximating spline functions for temperature within the length of each discrete element.

-Tashev Azat has implemented the formation of a functional of total thermal energy for all sampling elements, taking into account the boundary conditions when a heat flow enters one of the faces of a rectangular parallelepiped, and heat exchange with the environment occurs on the opposite side. And also, consideration of cases when the remaining faces of a rectangular parallelepiped are thermally insulated and vice versa.

-Aitbayeva Rakhatay has implemented the minimization of the function of the total thermal energy by the temperatures of the nodal points of a rectangular parallelepiped and obtaining resolving systems of linear algebraic equations to determine them.

-Kudaykulov Anarbay was responsible for solving and resolving systems of linear algebraic equations and obtaining the temperature value at the nodal points of a rectangular parallelepiped.

-Kunelbayev Murat solution determination of the temperature distribution law in a rectangular parallelepiped in accordance with the proposed temperature approximation formula.

-Mukaddas Arshidinova has developed algorithms for determining the temperature of the rod.

-Zhunusova Aliya solution development of programs in Python for the formation of general functionality, solving resolving systems of linear equations, and plotting the law of temperature distribution in a rectangular parallelepiped.

-Kazangapova Bayan has organized and executed the experiments of Section 4.

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Conflict of Interest

The authors have no conflict of interest to declare that is relevant to the content of this article.