Reliability and sensitivity analyses of structures related to Eurocodes

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Abstract—This article focuses on researching new concepts of global sensitivity analysis, which are directly oriented to reliability and the limit states of structures. A primary case study is performed to compare total sensitivity indices oriented to probability and design quantiles. The obtained results show that although the values of the total indices may differ, the sensitivity ranking is the same. Contrast functions are a suitable theoretical basis for sensitivity analysis. Reliability sensitivity analysis can be performed by following the concept of standard EN 1990 using design quantiles. The design quantiles of resistance and load are a suitable alternative to directly computing the probability of failure. Global sensitivity analysis oriented to design quantiles has proven helpful in measuring the influence of input variables on structural reliability.

Keywords—Sensitivity analysis, reliability, quantile, failure, failure probability, structure.

I. Introduction

THE research into the reliability of building structures is usually based on the probability of failure P_f and stochastic computational models [1, 2]. Alternative verification of structural reliability may be based on design quantiles as described in standard EN 1990 [3]. Estimating the design quantiles of resistance and load makes it possible to determine whether a design is reliable (when the resistance quantile is higher than the load quantile) or not (when the resistance quantile is lower than the load quantile).

The assessment of reliability using design quantiles does not require the direct calculation of P_f. The advantage of design quantiles is the quick and straightforward assessment of the reliability or unreliability of a structure based on quantile estimates using characteristic values and partial safety factors [4].

An essential supplement to structural reliability analysis is sensitivity analysis. Sensitivity analysis based on the outputs of engineering models differs in the subject of interest and employed computational methods, see, e.g., [5-9]. Traditional sensitivity analysis aims to determine how the variability of input parameters affects the output value [10]. The widely used approach of Sobol's sensitivity analysis is based on the decomposition of the output variance. Each member of the decomposition represents part of the contribution of the input (or group of inputs) to the output variance [11, 12]. Sobol's SA is very popular, and many researchers have applied Sobol's SA in their studies, see, e.g., [13-17].

However, research does not usually end with the procurement of the output as a random variable or histogram. Additional statistical estimates such as quantiles or P_f are needed to analyse structural reliability. Reliability-oriented sensitivity analysis (ROSA) is generally aimed at quantifying the importance of input variables to the structural reliability [18, 19], see also [20-23]. Furthermore, ROSA aimed at P_f analyses the influence of random input variables on the failure probability as the primary and most crucial measure of reliability [18, 19].

An alternative approach to the study of reliability is based on design quantiles, whose magnitudes and relationship to P_f are described in [3]. A research gap is global sensitivity analysis methods oriented to alternative reliability measures, such as design quantiles and the reliability index β [3].

Although the principles of reliability applied in [3] are well developed and historically verified, their application in ROSA is still a poorly managed task. Reliability assessment of standard [3] is based on design reliability conditions using design quantiles. By comparing the load and resistance quantiles, we can decide if the limit state has been reached or not, even though P_f is not estimated. The influence of random input variables on design quantiles can be studied using quantile-oriented sensitivity analysis (QOSA).

One type of QOSA is sensitivity analysis based on contrasts [24]. Contrast functions study variability according to the subject of interest, e.g., changes around the mean are significant for variance, changes around the quantile are significant for quantile, etc. Building on the contrast, new types of sensitivity analysis were introduced based on the quantile deviation or the square of the quantile deviation [25, 26]. A different approach is presented by the sensitivity measure based on the mean distance between quantiles and conditional quantiles [8] rather than the mean distance between average contrast functions, as in the case of quantile-oriented sensitivity indices.

It can be noted that QOSA has the sum of all indices equal to one [24]. QOSA can assign input variables a sensitivity ranking similar to Sobol's sensitivity analysis, even though the values of the first- and higher-order sensitivity indices can vary significantly. The differences are mainly in the values of the interaction effects, where contrast shows very high interactions compared to Sobol's sensitivity analysis. Although variance is one of the factors influencing the quantile, the quantile is influenced by the whole probability distribution of the model output with a significant influence of outliers and their interactions. Many of these and many other issues remain unresolved and require further analysis.

In structural mechanics applications, the use of total indices to study the sensitivity ranking of the effects of input variables on the quantile and comparisons with other methods have been published in [27, 28]. The main challenge of quantile-oriented sensitivity analysis is integrating it into the concept of limit states to make it a valuable tool for structural reliability analysis.

II. General Principles of Structural Reliability

The general principles of structural reliability are described in standard EN 1990 [1] and ISO 2394 [29]. Two types of limit states are typically verified: ultimate limit states and serviceability limit states. Reaching the limit state is a random phenomenon influenced by several significant uncertainties that depend on the nature of the structure, the environmental conditions and the applied activities. In general, the following types of uncertainties can be identified:

- Random nature of material properties, geometric characteristics and loads.
- Statistical uncertainties due to the limited size of the available data.
- Uncertainties of resistance and load effects in computational models due to the simplification of actual conditions.
- Vagueness due to inaccurate definitions of performance requirements.
- Gross errors in design, implementation and use.
- Lack of knowledge about the behaviour of new materials in real conditions.

Natural randomness and statistical uncertainties can be described using the available probability theory and methods of mathematical statistics [1, 2]. The basis of the probability theory is a continuous or discrete random variable whose value depends on a random event. The geometric and material characteristics of structures are generally continuous random variables. In contrast, the failure of the structure is a binary random variable that attains the value of 1 (failure) or 0 (success). The classic Monte Carlo method is the traditional method for estimating the model output's random realisations. By improving the Monte Carlo method, other efficient methods for estimating the probability of failure in engineering reliability have been developed [30]. However, the efficiency is usually obtained by strong assumptions as many methods trading generality for efficiency.

A well-established method with high efficiency is the first-order reliability method FORM [3], which analyses the reliability of a structure using the first two statistical moments of the model output, see, e.g., [31]. The Latin Hypercube Sampling method is usually used to efficiently estimate the first two statistical moments [32, 33]. Although the Monte Carlo method is numerically demanding, its use is experiencing a partial comeback due to increasing computing power. The method which improve efficiency with no significant loss in generality is stratified sampling [34, 35]. For instance, the article [36] proposes a new Monte Carlo-based method, which can be seen as a variant of stratified sampling. Metamodels and Response Surface methods present a separate area of research [30].

Although the uncertainties of computational models cannot be eliminated entirely, they can be assessed to some extent by theoretical and experimental research. The EN 1990 [3] standard provides the basic concept and techniques for analysing structural reliability. Although various types of uncertainties are becoming more significant, the standards [3, 29] do not include methods for quantifying the importance of all the different types of uncertainties. This article aims to introduce tools of sensitivity analysis, whose background concept techniques and theoretical bases can be used to quantify uncertainty.

III. Reliability Oriented Sensitivity Analysis

The aim of structural reliability analysis is the estimation of the theoretical failure probability, defined as:

$$P_f = P(Z \le 0). \tag{1}$$

where $Z = g(\mathbf{X})$ is a random variable called safety margin, which is a function of a random vector $\mathbf{X} = \{X_1, X_2, \ldots, X_n\}^T$, where n is the number of random input variables. It is assumed that variables $\mathbf{X}$ are statistically independent, a common assumption of global sensitivity analysis that builds on Sobol [24]. The global sensitivity analysis methods applied in this article are not limited in terms of the type of probability density functions (pdf) of $\mathbf{X}$ or the stochastic model used for the estimation of P_f. In engineering applications, P_f is usually a small value that must be lower than the target failure probability P_{ft} when reliability is of interest [23].

Probabilistic reliability analysis based on P_f quantifies reliability much more accurately than engineering analysis of reliability based on Eurocode standards. As a result, more load-bearing and reliable structures can be designed at a lower cost. However, these benefits are not free. The disadvantage is the high computational costs in optimisation analyses and the increased demand for input data of stochastic computational models, which can be guaranteed during the design and construction of the structure but may change during the use of the structure from the beginning to the end of its service life. Therefore, care should be taken in the design of structures, especially for those parameters that have a significant influence on reliability and, at the same time, may change during the structure's lifetime. Identifying these parameters is possible using ROSA oriented to P_f or design quantiles.

A. Sensitivity Analysis of Failure Probability

In structural reliability analysis, sensitivity analysis can be used to measure how the input variable influences the failure of a structure [5]. ROSA oriented to P_f was introduced in [18, 19]. The sensitivity indices oriented to P_f are based on the decomposition of the variance $V(1_{Z<0})$ of a binary function (failure, success) building on Sobol decomposition [11, 12], where the sum of all indices is equal to one and each index is non-negative.

The first-order probability contrast index C_i is defined in [24]. For practical use, the first-order probability contrast index C_i can be written in the form first introduced in [22]:

$$C_i = \frac{P_f\left(1-P_f\right) - E\left(P_f|X_i\left(1-P_f|X_i\right)\right)}{P_f\left(1-P_f\right)}, \tag{2}$$

where the mean value E[·] is considered over all random realisations of X_i and $P_f(1- P_f)$ is the variance of the Bernoulli distribution of the binary outcome: success (0) or failure (1). The same indices can be obtained using the contrast function [24], where P_f is a minimizer. The second-order sensitivity index C_{ij} can be calculated similarly using the fixation of two random input variables:

$$C_{ij} = \frac{P_f\left(1-P_f\right) - E\left(\left(P_f|X_i, X_j\right)\left(1-P_f|X_i, X_j\right)\right)}{P_f\left(1-P_f\right)} - C_i - C_j \cdot \tag{3}$$

Additional sensitivity indices, which quantify higher-order interaction effects, can be formulated similarly. Examples demonstrating the rationality of P_f-oriented sensitivity indices have been presented in many engineering applications, e.g., [37-39]. The sensitivity indices are either positive or zero. The sum of all sensitivity indices must be equal to one.

Estimating all 2^n-1 sensitivity indices can be computationally demanding, and the effects of the individual input variables may be dispersed in numerous interaction effects. Due to the simple determination of the sensitivity ranking, it is more practical to calculate the total indices that quantify the influence of variable X_i, including all interaction effects with the other variables.

$$C_{Ti} = 1 - \frac{P_f\left(1-P_f\right) - E\left(\left(P_f|X_{\sim i}\right)\left(1-P_f|X_{\sim i}\right)\right)}{P_f\left(1-P_f\right)} \cdot \tag{4}$$

where $X_{\sim i}$ denotes input random variable X_i and fixed variables ($X_1, X_2, \ldots, X_{i-1}, X_{i+1}, \ldots, X_N$).

Equations (2), (3) and (4) are directly oriented to P_f. Therefore, the sensitivity analysis conceived in this manner directly quantifies the influence of random input variables on reliability, measured by the value of P_f.

B. Sensitivity Analysis of Quantile

Quantile-oriented sensitivity analysis examines changes around the α-quantile of output Y caused by changes in input variables $\mathbf{X} = \{X_1, X_2, \ldots, X_n\}^T$. Changes around the α-quantile are quantified by the contrast function ψ.

$$\psi(\theta) = E\left(\psi(Y,\theta)\right) = E\left(\left(\alpha - 1_{R<\theta}\right)\left(Y-\theta\right)\right). \tag{5}$$

where Y is a scalar output. Equation (5) becomes a sensitivity measure if the argument θ has a value equal to α-quantile.

$$\theta^* = \underset{\theta}{\mathrm{Argmin}}\, \psi(\theta) = \underset{\theta}{\mathrm{Argmin}}\, E\left(\left(\alpha - 1_{R<\theta}\right)\left(Y-\theta\right)\right) \cdot \tag{6}$$

where θ^* is equal to the α-quantile of Y. When θ^* is used, the contrast function ψ reaches its minimum, which can be written in the form using the quantile deviation l:

$$\psi\left(\theta^*\right) = E\left(\left(\alpha - 1_{R<\theta^*}\right)\left(Y-\theta^*\right)\right) = l \cdot \alpha \cdot \left(1-\alpha\right). \tag{7}$$

where l is α-quantile deviation [25, 26] of model output Y with probability density function $f(y)$.

$$l = \underbrace{-\frac{1}{\alpha}\int_{-\infty}^{\theta^*} y \cdot f(y)dy}_{Subquantile} + \overbrace{\frac{1}{1-\alpha}\int_{\theta^*}^{\infty} y \cdot f(y)dy}^{Superquantile} \cdot \tag{8}$$

The quantile deviation l is defined as the difference between superquantile $E(Y|Y \geq \theta^*)$ and subquantile $E(Y|Y < \theta^*)$. The quantile deviation l was first introduced as a sensitivity measure in [25, 26]. The first-order probability contrast index Q_i based on the quantile deviation l can be written as:

$$Q_i = \frac{l - E\left(l|X_i\right)}{l}, \tag{9}$$

where the mean value E[·] is considered over all random realisations of X_i. The second-order α-quantile sensitivity index Q_{ij} is formulated similarly by fixing pairs X_i, X_j.

$$Q_{ij} = \frac{l - E\left(l|X_i, X_j\right)}{l} - Q_i - Q_j, \tag{10}$$

where E[·] is considered across all random realisations of X_i and X_j. The sum of all sensitivity indices must be equal to one.

The influence of variable X_i, including all interaction effects with other variables, is quantified by the total index Q_{Ti}.

$$Q_{Ti} = 1 - \frac{l - E\left(l \middle| X_{\sim i}\right)}{l}. \tag{11}$$

where $X_{\sim i}$ denotes input random variable X_i and fixed variables ($X_1, X_2, \ldots, X_{i-1}, X_{i+1}, \ldots, X_N$).

Comparing sensitivity indices Q_i, Q_{ij}, etc., with classical Sobol's SA indicates the difference in the quantile deviation l and variance unit. It can be shown that replacing the quantile deviation l with its square is essential in the search for a sensitivity measure useable in engineering tasks.

The first-order probability contrast index K_i based on l^2 (square of quantile deviation) can be formulated as:

$$K_i = \frac{l^2 - E\left(\left(l \middle| X_i\right)^2\right)}{l^2}, \tag{12}$$

where the mean value E[·] is considered over all random realisations of X_i. The second-order sensitivity index K_{ij} can be formulated by fixing pairs X_i and X_j.

$$K_{ij} = \frac{l^2 - E\left(\left(l \middle| X_i, X_j\right)^2\right)}{l^2} - Q_i - Q_j, \tag{13}$$

where E[·] is considered across all random realisations of X_i and X_j. The sum of all sensitivity indices is equal to one. The influence of variable X_i, including all interaction effects with other variables, is quantified using the total index Q_{Ti}, which can be formulated as:

$$K_{Ti} = 1 - \frac{l^2 - E\left(\left(l \middle| X_{\sim i}\right)^2\right)}{l^2}. \tag{14}$$

where $X_{\sim i}$ denotes input random variable X_i and fixed variables ($X_1, X_2, \ldots, X_{i-1}, X_{i+1}, \ldots, X_N$).

Total order sensitivity indices take into account both the main, second order and higher order effects, which involves the evaluation over a full range of input parameter space. Although the sensitivity indices of the first, second and higher orders are essential for a complete description of all main and interaction effects, total indices C_{Ti}, Q_{Ti}, and K_{Ti} are of more practical use as they facilitate the determination of the sensitivity ranking at a reasonable computational cost.

IV. Numerical Example

This section presents a stochastic computational model of a load-bearing steel member subjected to axial tension. The subject of the analysis is the ultimate limit state. A steel bar is considered without failure if the difference between resistance R and load E is greater than zero:

$$Z = R - E. \tag{15}$$

Load action E consists of two load cases, E_1 and E_2, with Gauss pdf; see Table I.

$$E = E_1 + E_2. \tag{16}$$

Both load cases are considered statistically independent. Since E_1 and E_2 have Gauss pdf, product E also has a Gauss pdf.

TABLE I
Input Random Variables of Load Action

Characteristic	Pdf	Mean value	St. deviation
Load action E_1	Gauss	201.604 kN	24.2 kN
Load action E_2	Gauss	28.364 kN	23.9 kN

The resistance R is a random variable with the unit kN. The resistance is a product of the yield strength f_y and cross-sectional area. The cross-section has a rectangular shape with dimensions a and b, see Table II.

TABLE II
Input Random Variables of Resistance

Characteristic	Pdf	Mean value	St. deviation
Dimension a	Gauss	100 mm	1 mm
Dimension b	Gauss	10 mm	0.46 mm
Yield strength f_y	Gauss	412.68 MPa	27.941 MPa

The resistance R can be calculated as the product of the cross-sectional area of the rectangle $a \cdot b$ and yield strength f_y.

$$R = a \cdot b \cdot f_y. \tag{17}$$

The probability density function of R can be approximated by a three-parameter lognormal pdf using the mean value, standard deviation and standard skewness, which have been analytically derived in [25]. The mean value μ_R, standard deviation σ_R and standard skewness of R are μ_R = 412.7 kN, σ_R = 34.1 kN and 0.11. Using a three-parameter lognormal pdf, P_f can be computed from (1) using the integral

$$P_f = \int_{-\infty}^{\infty} \Phi_R(y) \varphi_E(y) \mathrm{d}y. \tag{18}$$

where $\varphi_E(y)$ is a Gauss pdf of load action, $\Phi_R(y)$ is a three-parameter lognormal distribution function of R. The value of the integral is estimated numerically following the algorithms for practical calculation published in [25, 40].

The random input variables are chosen so that the standard deviation of E is approximately the same as the standard deviation of R. This is so that the results of the sensitivity analysis of P_f can be compared with the results of the sensitivity analysis of the design quantiles. The general cases of the standard deviations of load and resistance and their possible influence on the sensitivity analysis results are discussed in the following chapters.

V. The Results of Sensitivity Analysis

Sensitivity indices of P_f are computed numerically using

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integration (18). Integration in (18) is performed by Simpson's rule, employing more than ten thousand integration steps over the interval $[\mu_Z-10\sigma_Z, \mu_Z+10\sigma_Z]$ of the limit state function (15). The mean values E[·] in (2), (3) and (4) are computed numerically using algorithms of numerical integration [25, 40].

Figure 1 shows the results of the sensitivity analysis of P_f, which quantify the influence of five input variables, E_1, E_2, a, b, and f_y, using all main and interaction effects. This calculation is complete but tedious, and the effects of the input variables are dispersed in the interaction effects. Therefore, it is better to determine the sensitivity ranking by calculating and analysing the magnitudes of the total indices. Total indices quantify the contribution of X_i, including all effects caused by its interactions, of any order, with any other input variables. The greater the value of the total index, the greater the influence of the input variable on the studied output (P_f or quantile).

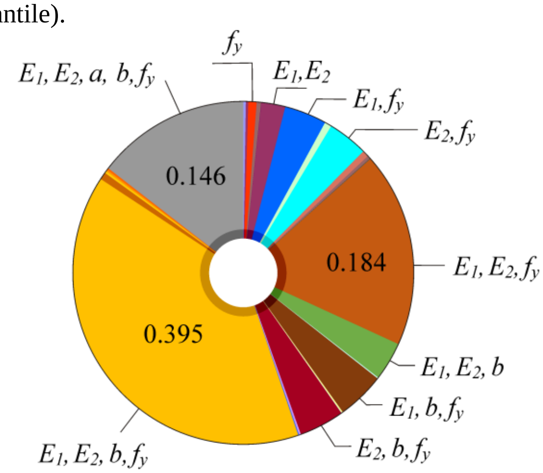

Fig. 1 Global sensitivity analysis of P_f – all indices.

The total indices can be obtained by determining the sensitivity indices of all orders or directly using (4), see Fig. 2. The direct calculation using (4) is more efficient because it does not require calculating the first- and higher-order indices but only the total indices are sufficient to determine the sensitivity ranking. Figure 2 shows that the sensitivity ranking determined from the sensitivity analysis of P_f is f_y, E_1, E_2, b, a. Although f_y is the dominant variable, the effects of E_1 and E_2 are comparable to the effect of f_y.

Quantile-oriented sensitivity analysis was computed for 0.9963-quantile of E and 0.001-quantile of R [25]. If $E < E_d$ and $R > R_d$, the failure probability is smaller than 7.2E-5 [3].

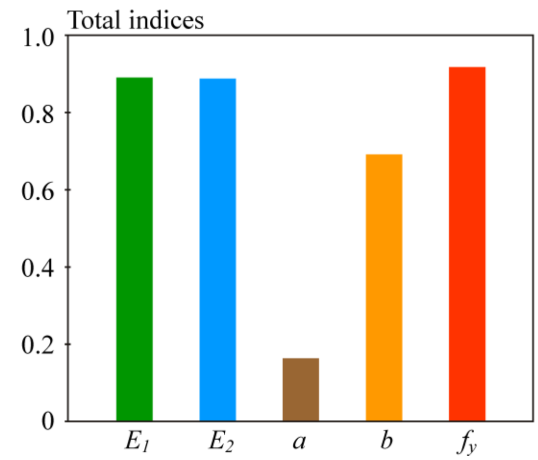

Fig. 2 Global sensitivity analysis of $P_f - C_{Ti}$ indices.

Figure 3 shows the total indices Q_{TI} computed according to (11). Figure 4 shows the total indices K_{TI} computed according to (14). Total indices Q_{TI} and K_{TI} are quantile-oriented, so the load effect is analysed independently of the resistance.

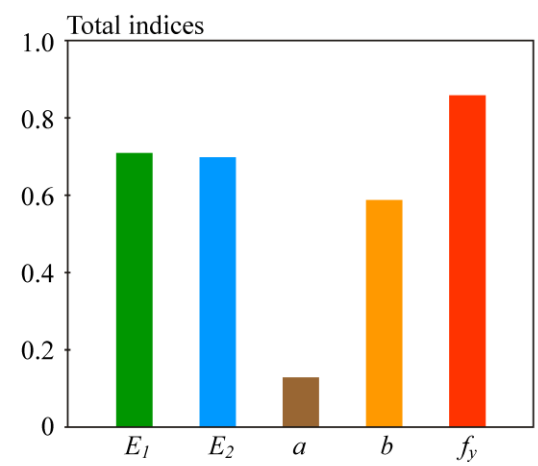

Fig. 3 Global sensitivity analysis of $P_f - Q_{Ti}$ indices.

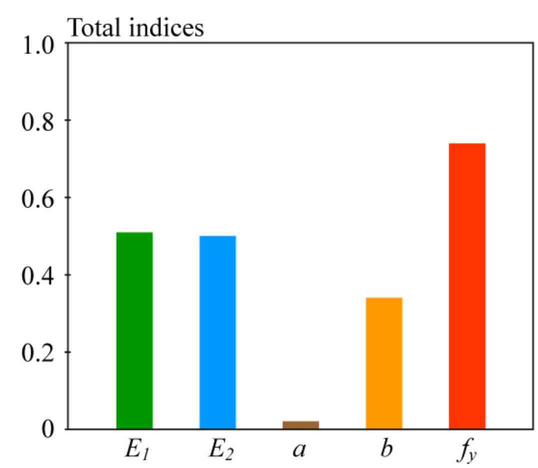

Fig. 4 Global sensitivity analysis of $P_f - K_{Ti}$ indices.

Although the total indices vary, the ranking of the input variables from the most influential to the least influential is always the same: f_y, E_1, E_2, b, a. The dominant variable is the

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yield strength f_y. The second dominant variables are the load effects E_1 and E_2. The variability of random variables a and b has the most negligible effect on the reliability. Neglecting the random variability of a has a negligible effect on P_f or the quantile, which was verified numerically.

VI. Discussion

The concept of limit states is based on the separate effect of resistance and load, making it possible to evaluate both quantities separately. As a result, reliability can be assessed by comparing the design quantiles of resistance and load more efficiently than by directly calculating P_f.

The research aims to find a comprehensive concept of global sensitivity analysis with the separate analysis of load and resistance. Although the loss of interaction effects between load cases and input variables influencing the resistance is not yet resolved, the case study results suggest the possibility of effectively using total sensitivity indices to identify the sensitivity ranking. Cross interactions between the load and resistance may not be meaningful when determining the sensitivity ranking if total indices are used and the load and resistance variances have the same value, see Fig. 5. Then, the sensitivity ranking of the design quantiles is the same as the sensitivity ranking determined using the sensitivity analysis of P_f.

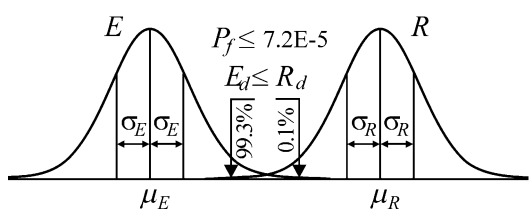

Fig. 5 Limit state design based on design quantiles.

In general, the variance of the load may be more or less different from the resistance. The sensitivity analysis of P_f based on the decomposition of the variance $V(1_{Z<0})$ quantifies the influence of input variables on the load and resistance side, including all interaction effects between input variables. The total indices of the design quantile can be corrected by weights based on the FORM sensitivity factors α_E, α_R of the standard [3].

$$\alpha_E = \frac{\sigma_E}{\sqrt{\sigma_E^2 + \sigma_R^2}}, \alpha_R = \frac{\sigma_R}{\sqrt{\sigma_E^2 + \sigma_R^2}}, \text{ with } \alpha \leq 1. \tag{19}$$

For example, suppose the standard deviation of the load prevails. In that case, the total indices of the load are increased by the appropriate weighting factor α_E, where $\alpha_E > \alpha_R$. Another example, if (theoretically) the standard deviation of resistance was equal to zero, the total indices of resistance are equal to zero because they are multiplied by weighting factor $\alpha_E = 0$. A detailed concept of weighting factors can be proposed based on their application in global sensitivity analysis, see, e.g., [25].

In practical applications, the reliability of structures with many load cases is often addressed. The decision-maker needs some indication of how sensitive the choices will be to changes in one or more inputs [41, 42]. For reliability models, the sensitivity analysis shows which load cases and their combinations significantly influence the reliability and which structural elements can be overloaded. For more complex structural models, the effects of other structural properties, such as boundary conditions, damping properties, material constants and geometric parameters, on the static and dynamic structural responses can be analysed. The goal of practical applications is an optimal design that seeks a compromise between high reliability and low cost of the load-bearing structure.

It can be noted that the concept of design conditions of reliability of the standard [3] using sensitivity factors α_E, α_R is based on the assumption of Gauss pdf of load and resistance and, therefore, cannot be fully applied to extreme distributions such as Gumbel or Weibull pdf. Nevertheless, the design conditions of reliability do not strictly require Gauss pdf, but other shapes close to the Gauss pdf are tolerated and can be considered Gauss-like in engineering applications [43, 44]. The shape of the pdf of resistance usually deviates a little from the Gauss pdf due to small skewness, but there may be more significant differences in the load. Gumbel or Weibull pdf is more common for modelling short-term and long-term single variation actions, while permanent load action or resistance is usually modelled using Gauss pdf [45].

On the one hand, indices oriented to P_f or design quantiles are easy to interpret, making them a valuable tool for analysing structural reliability. However, on the other hand, their formulations include multiple integrals, the estimation of which using numerical integration or Monte Carlo (or quasi-Monte Carlo) methods requires many model simulations in practice, which considerably limits their use in the case of expensive models [46, 47]. To overcome this drawback, effective estimation strategies based on the use of metamodels based on polynomial chaos expansions [48, 49] or artificial neural networks [50, 51] have been proposed in literature.

Although sensitivity indices oriented to P_f are computationally more time-consuming, most of the time needed for the numerical estimation of P_f can be saved because reaching the failure domain of $1_{Z<0}$ can be considered a consequence of a specific combination of inputs that is predictable. In contrast, quantile-oriented sensitivity indices are less time-consuming, but not as many efficient strategies have been developed to speed up the estimation of the quantile deviation because this variable is dependent on the shape of the entire distribution (the absolute difference between two average values of the population before and after the quantile). Furthermore, the case study [26] showed that the correlation between the standard deviation and the quantile deviation may or may not be high. In general, quantile-oriented sensitivity indices differ more or less from Sobol's sensitivity indices,

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where the results of quantile-oriented sensitivity analysis have a significantly higher proportion of interaction effects than Sobol [27, 28].

Other sensitivity analysis methods may be more or less empathetic to the reliability, even though neither the design quantile nor P_f is the subject of their interest [22]. For example, estimating the sensitivity ranking can be determined based on the correlation between the input and output [52], which do not require double-loop simulations and are therefore directly available as a by-product of the statistical analysis of the output using Monte Carlo methods. For the case study presented here, the correlation between input random variable X_i and output random variable Z is shown in Fig. 6.

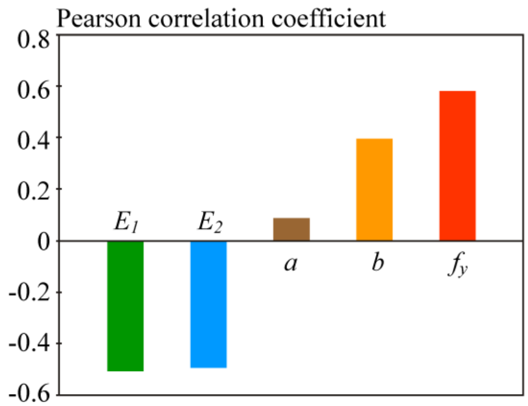

Fig. 6 Correlation coefficient between Z and X_i.

The sensitivity ranking according to Fig. 6 is the same as according to the total indices in Fig. 2 to Fig. 4. The correlation coefficients do not analyse the interaction effects but provide information about the positive or negative influence of the input variable on the model output, see Fig. 7 and Fig. 8.

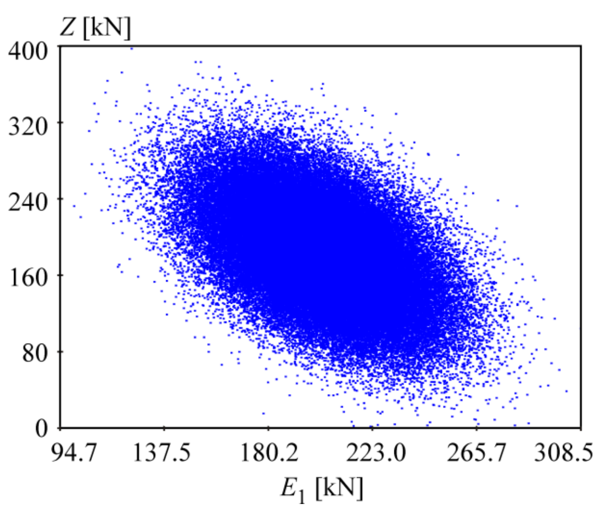

Fig. 7 Correlation dependence between Z and E_1 (X_1).

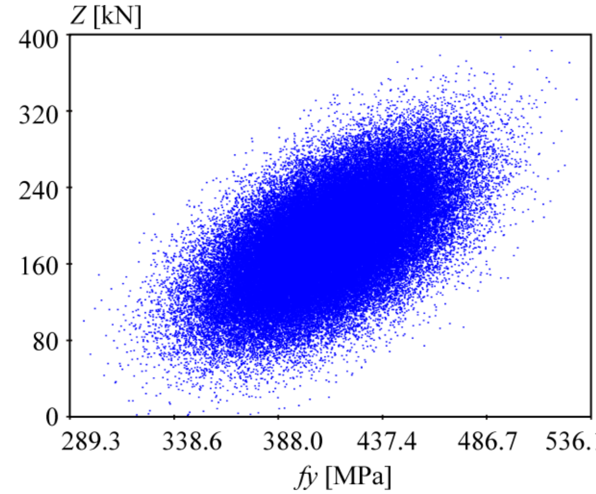

Fig. 8 Correlation dependence between Z and f_y (X_5).

Sensitivity analysis based on the correlation of the model output with the inputs has certain limitations, consisting of the condition of monotonic dependence between the input and the output. Correlation implies dependence, but the opposite is not true. In general cases, monotonic dependencies between inputs and outputs cannot be guaranteed for black-box type computational models such as those based on the finite element method using non-linear analysis, see, e.g., [53-55]. A non-monotonic relationship between input and output may give a weak or zero correlation even though the effect of the input on the output is significant [56, 57].

Figure 9 shows an example of zero correlation corr(R, e_0) = 0 for a case study with a strong functional dependence between the initial amplitude of the axial curvature e_0 and the static resistance of a slender column subjected to vertical compressive load R.

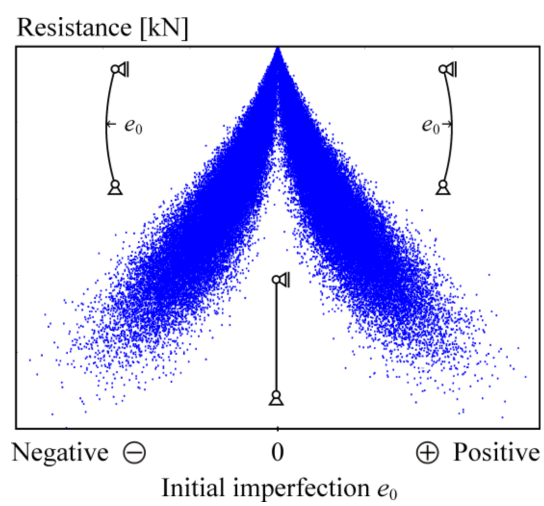

Fig. 9 Resistance of compressed column vs initial imperfection.

The maximum elastic resistance at the Euler critical force level occurs for $e_0 = 0$, see Fig. 9. The elastic resistance decreases if e_0 decreases and $e_0 < 0$, but the elastic resistance also decreases if e_0 increases and $e_0 > 0$. Although correlations are a direct output of the statistical analysis of the model output, the results of a sensitivity analysis based on correlations can be misleading.

In some instances, the relationship between input and output can be changed or supplemented to achieve monotonicity without affecting the statistics of the model output. For the example shown in Figure 9, a monotonic dependence can be achieved by introducing the absolute value of $|e_0|$, but with a restriction to cross-sections symmetric to the axis about which bending due to buckling occurs. The value of corr(R, $|e_0|$) then expresses the sensitivity between the amplitude of initial curvature $|e_0|$ and resistance R correctly. The example shows that the sensitivity analysis results based on the correlation coefficients must be interpreted cautiously only after a previous analysis of the monotonic dependence between the output with verification for each input variable.

Sensitivity analysis based on the model output variance is very popular in basic sciences [10], although the variance is only empathetic to reliability. The influential and non-influential variables can also be identified by analysing the distribution shape [58] or neural network ensemble-based sensitivity analysis [59]. However, change is not directly studied by these methods.

Sensitivity indices directly oriented to P_f are based on $V(1_{Z<0})$, where the binary assumption $1_{Z<0}$ leads to a loss of information about the distance from the critical boundary $Z = 0$. It can be discussed whether all failures have the same weight (e.g., loss of stability of very slender columns) or whether failures with a greater distance from Z can have greater weight (relative failure). Loss of stability is a fatal failure leading to the collapse of an entire system in the case of statically determinate structures. However, a statically indeterminate structure can survive the buckling of one member with the support of the rest of the system. Similarly, exceeding the yield strength at one critical point does not necessarily mean the collapse of the entire system when the material has a plastic reserve, as is the case with lower-grade carbon steel. In the case of the serviceability limit state, a small deformation over the limit is less severe than a large deformation over the limit. Although it is common to model failure using a discrete random variable (1 failure, 0 success), limit states could be studied using an alternative concept that more appropriately considers the character of failure from temporary to permanent, from minor to severe and very critical. The question of suitable methods for studying relative failures and their combinations using global sensitivity analysis remains unanswered.

VII. Conclusion

Sensitivity analysis is one of the tools that help decision-makers do more than just solve an engineering problem. Sensitivity analysis provides valuable insight into the issues associated with the computational model. The decision-maker finally gets a decent idea of how sensitive the chosen optimal solution is to possible changes in the input values of one or more parameters.

This article introduced the concept of global sensitivity analysis oriented to conditional failure probability and design quantiles. The case study showed that reliability-oriented sensitivity analysis could use a common platform based on the concept of limit states and design quantiles. Sensitivity analysis of design quantiles successfully competes with sensitivity analysis of failure probability, although certain details must be addressed. The sensitivity analysis results of the design quantiles show the same sensitivity ranking of input variables as the sensitivity analysis of the failure probability if total indices are used.

Although the same standard deviation of load and resistance was assumed in the case study, possible approaches for more general applications (with different standard deviations of load and resistance) were presented. Quantile-based sensitivity analysis has some advantages. Estimates of design quantiles are numerically less demanding than estimates of failure probability. Thus, the proposed concept may have promising applications, especially in tasks with numerically demanding computational models.

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