Mathematical modelling for studying the influence of the initial stress and relaxation time on reflection waves in thermo-piezoelectric half-space

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Abstract—The aim of this study is to illustrate of the influence of the thermal relaxation time according the Lord-Shulman model and the initial stress on the reflection of plane waves in an elastic transversely isotropic thermo-piezoelectric half-space. The governing equation and suitable boundary conditions of a transversely isotropic thermo-piezoelectric medium are solved to obtain reflection coefficients when incident waves are falling at the interface between vacuum and half-space. The effects of thermal relaxation time and the initial stress are calculated numerically and shown graphically on these coefficients

Keywords—Reflection coefficients; Thermo-piezoelectricity; Thermal relaxation time; Lord-Shulman model; Cadmium Selenide; Hexagonal crystals.

Nomenclature

$E_i = -\varphi_{,i}$ is the electric field,
u_i, φ, and T are the mechanical displacement, electric potential and absolute temperature,
D_i is the electric displacement,
$\sigma_{ij}, \sigma^o_{kj}$ are the stress and initial stress tensors,
ε_{ij} is the strain,
γ_{ij} is the thermal elastic coupling tensors
ρ is the density of the medium,
C_{ijkl} is the elastic parameters tensor,
e_{ijk} is the piezoelectric constants,
P_{ij} is the dielectric moduli,
d_i is the pyroelectric moduli,
t_o are thermal relaxation times,
K_{ij} is the heat conduction tensor,
T_o is the reference temperature,
C_E are the specific heat at constant strain,
δ_{ik} is the Kronecker delta

I. INTRODUCTION

Piezo-thermoelasticity theories, which admit a finite speed for thermal signals, have been receiving a lot of attention for the past four decades. The theory of thermo-piezoelectricity was first proposed by Mindlin[1]. The physical laws for the thermo-piezoelectric materials have been explored by Nowacki [2]-[4].Chandrasekharaiah[5],[6] has generalized Mindlin's theory of thermo-piezoelectricity to account for the finite speed of propagation of thermal disturbances. Various authors have worked on wave propagation in isotropic thermo-piezoelectricity. For example, Sharma and Kumar [7],Sharma and Walia[8],[9], Alshaikh[10].

The problem of reflection of plane waves at a plane interface of piezoelectric and thermoelasticity media has been discussed by many authors, Abd-alla and Al-dawy[11] discussed the reflection phenomena of *SV* waves in a generalized thermoelastic medium, Sharma et. al. [12] reflection of generalized thermoelastic waves from the boundary of a half-space.The reflection waves in pyroelectric and Piezoelectric Materials investigated by Kuang and Yuan [13], Alshaikh[14] studied the reflection transverse waves in the Green- Lindsay theory for kind of smart materials under initial stress and relaxation times effect. The effect of the initial stresses on the reflection and transmission of plane wave propagation are illustrated in many recent contributions such as [15]-[19].

In the present contribution, reflection of plane waves under initial stress of a hexagonal thermo-piezoelectric solid half-space is studied. Reflection coefficients of various reflected waves are obtained and illustrated numerically for a particular model to analyze effects of initial stressand thermal relaxation.

II. GOVERNING EQUATIONS

Following Lord and Shulman [20] and Sharma and Walia[8], the constitutive relations and hexagonal thermopiezoelectric equations under initial stress and relaxation time effect for two dimensions motion are given by:
Equation of motion and Gauss's equation

$$\left.\begin{aligned}\sigma_{ij,j} + \left(u_{ik}\sigma^o_{kj}\right)_{,j} &= \rho\ddot{u}_i,\\ D_{i,i} &= 0\end{aligned}\right\}(1)$$

Heat conduction equation

$$K_{ij}T_{,ij} = T_o\left[\gamma_{ij}\left(\dot{u}_{i,j} + t_o\delta_{ik}\ddot{u}_{i,j}\right) - d_{ij}\left(\dot{\varphi}_{,i} + t_o\delta_{ik}\ddot{\varphi}_{,j}\right)\right]$$

$+\rho C_E(\dot{T}+\delta_{ik}\ddot{T})$(2)

The strain-displacement relation and the electric field according to the quasi-static approximation have the forms as:

$$\left.\begin{aligned}\varepsilon_{ij}&=\tfrac{1}{2}(u_{i,j}+u_{j,i}),\\ E_i&=-\varphi_{,i}\end{aligned}\right\} i,j=1,2,3.(3)$$

Stress-strain-temperature and electric field relations

$$\sigma_{ij}=C_{ijkl}\varepsilon_{kl}-e_{kij}E_k-\gamma_{ij}T \quad (4)$$

The relation between the electric displacement, strain, electric field, and temperature

$$D_i=e_{ikl}\varepsilon_{kl}+P_{ik}E_k+d_iT \quad (5)$$

III. SOLUTION OF THE PROBLEM FOR INCIDENT *QP*-WAVE

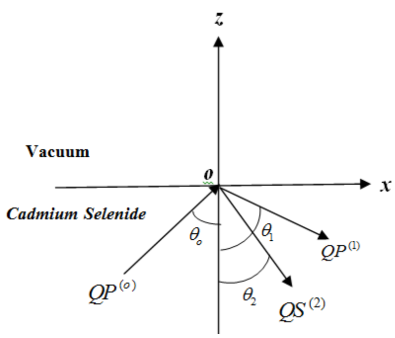

Figure 1

We consider a thermopiezoelectric plane wave *QP*propagating under initial stress through the medium, which we identify as the region $z\le 0$ and falling at the plane z = 0, with its direction of propagation making an angle θ_o with the normal to the surface. Corresponding to incident wave, we get the waves in mediumas reflected *QP*-, *QS*-, φ- and T- waves. The complete geometry of the problem is shown in Figure 1. Let the wave motion in this medium be characterized by the displacement $\vec{u}(u,0,w)$, the temperature T vector, and the electric potential functionφ, all these quantities being dependent only on the variablesx,z,t. We assume solution of the form (Achenbach [21]):

$(u^{(o)},w^{(o)},\varphi^{(o)},T^{(o)})=(A_o\sin\theta_o,A_o\cos\theta_o,B_o,C_o)\,exp[\vartheta_o]$

$(u^{(1)},w^{(1)},\varphi^{(1)},T^{(1)})=$
$(A_1\sin\theta_1,-A_1\cos\theta_1,B_1,C_1)\,exp[\vartheta_1]$

$(u^{(2)},w^{(2)},\varphi^{(2)},T^{(2)})=(A_2\cos\theta_2,A_2\sin\theta_2,B_2,C_2)\,exp[\vartheta_2]$
(6)

where

$\vartheta_0=ik_o(x\sin\theta_o+z\cos\theta_o-C_{Lo}t)$,

$\vartheta_1=ik_1(x\sin\theta_1-z\cos\theta_1-C_{L1}t)$,

$\vartheta_2=ik_2(x\sin\theta_2-z\cos\theta_2-C_{T2}t)$,

Also, here $C_{L0}=\omega/k_0,C_{L1}=\omega/k_1,C_{T2}=\omega/k_2$are the velocity of incident *QP*, reflected *QP*, reflected *QS* wave.

IV. THE BOUNDARY CONDITIONS

The free mechanical boundary conditions

$$\sigma_{zx}^{(O)}+\sigma_{zx}^{(1)}+\sigma_{zx}^{(2)}=0 \quad (7)$$

$$\sigma_{zz}^{(O)}+\sigma_{zz}^{(1)}+\sigma_{zz}^{(2)}=0 \quad (8)$$

The electrical condition

$$\varphi^{(O)}+\varphi^{(1)}+\varphi^{(2)}=0 \quad (9)$$

The thermal condition

$$T^{(O)}+T^{(1)}+T^{(2)}=0 \quad (10)$$

Using equations (3-4), and (6) of hexagonal (6 mm) crystals into equations (7)-(10), we obtain the following set of equations:

$$ik_o[C_{44}\sin 2\theta_o\,A_o+e_{15}\sin\theta_o\,B_o]\exp[\vartheta_0]$$
$$-ik_1[C_{44}\sin 2\theta_1\,A_1-e_{15}\sin\theta_1\,B_1]\exp[\vartheta_1]$$
$$-ik_2[C_{44}\cos 2\theta_2\,A_2-e_{15}\sin\theta_2\,B_2]\exp[\vartheta_2]=0(11)$$

$$ik_o[(C_{13}\sin^2\theta_o+C_{33}\cos^2\theta_o)A_o+e_{33}\cos\theta_oB_o-(\gamma_3/ik_o)C_o]\exp[\vartheta_o]-ik_1[(C_{13}\sin^2\theta_1+C_{33}\cos^2\theta_1)A_1-e_{33}\cos\theta_1B_1-(\gamma_3/ik_1)C_1]\exp[\vartheta_1]+ik_2[(C_{13}-C_{33})sin\theta_2\cos\theta_2A_2-e_{33}\cos\theta_2B_2-(\gamma_3/ik_2)C_2]\exp[\vartheta_2]=0 \quad (12)$$

$$B_oexp[\vartheta_o]+B_1\,exp[\vartheta_1]+B_2\,exp[\vartheta_2]=0(13)$$

$$C_oexp[\vartheta_o]+C_1\,exp[\vartheta_1]+C_2\,exp[\vartheta_2]=0(14)$$

Equations (11-14) must be valid for all values of $t$and x, hence

$$\left.\begin{aligned}\vartheta_o=\vartheta_1=\vartheta_2,\\ k_o\sin\theta_o=k_1\sin\theta_1=k_2\sin\theta_2,\\ k_oC_{Lo}=k_1C_{L1}=k_2C_{T2}=\omega\end{aligned}\right\} \quad (15)$$

From the above relations, we get

$$\left.\begin{aligned}k_o=k_1,\theta_o=\theta_1,C_{Lo}=C_{L1},\\ k_2/k_o=C_{Lo}/C_{T2}=\tau,\\ \sin\theta_2=\sin\theta_o/\tau\end{aligned}\right\}(16)$$

Furthermore, we should now use the equations (6) when z=0 of the media, i.e., using equations (1-2) which will give us additional relations between amplitudes

$$\left.\begin{aligned}\chi_oA_o+R_oB_o+\mu_oC_o=0,\\ \chi_1A_1+R_1B_1+\mu_1C_1=0,\\ \chi_2A_2+R_2B_2+\mu_2C_2=0\end{aligned}\right\}(17)$$

$$\left.\begin{aligned}L_oA_o+G_oB_o+S_oC_o=0,\\ L_1A_1+G_1B_1+S_1C_1=0,\\ L_2A_2+G_2B_2+S_2C_2=0\end{aligned}\right\}(18)$$

$$\left.\begin{aligned}E_oA_o+D_oB_o+F_oC_o=0,\\ E_1A_1+D_1B_1+F_1C_1=0,\\ E_2A_2+D_2B_2+F_2C_2=0\end{aligned}\right\}(19)$$

where

$\chi_o=-\sin\theta_o[\rho C_{Lo}^2-(C_{11}+\sigma_{xx}^o)\sin^2\theta_o-(C_{13}+2C_{44}+\sigma_{zz}^o)\cos^2\theta_o]$

$R_o=(e_{13}+e_{15})\sin\theta_o\cos\theta_o,\ \ \mu_o=-i\gamma_1\sin\theta_o/k_o$

$\chi_1=-\chi_o,\qquad R_1=R_o,\qquad \mu_1=-\mu_o,$

$\chi_2=\cos\theta_2[\rho C_{T2}^2-(C_{11}+\sigma_{xx}^o)\sin^2\theta_2-(C_{44}+\sigma_{zz}^o)\cos^2\theta_2+(C_{13}+C_{44})\sin^2\theta_2]$

$R_2=(e_{13}+e_{15})\sin\theta_2\cos\theta_2,\ \ \mu_2=-i\gamma_1\sin\theta_2/k_2,$

$L_o=-[(e_{13}+2e_{15})\sin^2\theta_o\cos\theta_o+e_{33}\cos^3\theta_o]$,

$G_o=P_{11}\sin^2\theta_o+P_{33}\cos^2\theta_o,\ S_o=id_3\cos\theta_o/k_o,$

$L_1=-L_o,\quad G_1=G_o,\ S_1=-S_o,$

$L_2=[(e_{13}+e_{15}-e_{33})\sin\theta_2\cos^2\theta_2-e_{15}\sin^3\theta_2]$

$G_2=P_{11}\sin^2\theta_2+P_{33}\cos^2\theta_2,\ S_2=-id_3\cos\theta_2/k_2$

$E_o=T_o(1-ik_ot_o\delta C_{Lo})(\gamma_1\sin^2\theta_o+\gamma_3\cos^2\theta_o),$

$D_o=-T_od_3(1-ik_ot_o\delta C_{Lo})\cos\theta_o,$

$F_o=[(K_1\sin^2\theta_o+K_3\cos^2\theta_o)/C_{Lo}]-[\rho iC_E(1-ik_ot_oC_{Lo})/k_o]$

$E_1=-E_o,\quad D_1=D_o,\ F_1=-F_o,$

$E_2=-T_o(1-ik_2t_o\delta C_{T2})(\gamma_1-\gamma_3)\sin\theta_2\cos\theta_2,$

$D_2=-T_od_3(1-ik_2t_o\delta C_{T2})\cos\theta_2,$

$F_2=[\rho iC_E(1-ik_2t_oC_{T2})/k_2]-[(K_1\sin^2\theta_2+K_3\cos^2\theta_2)/C_{T2}]$

Solving equations (17-19), we can determine the reflection coefficients as:

$A_1/A_o=(\eta_1+\eta_2)/(\eta_1-\eta_2)$, (20)

$A_2/A_o = 2/(\eta_2 - \eta_1)$(21)
$B_1/B_o = -A_1/A_o$ (22)
$B_2/B_o = (A_2/A_o - 1)$ (23)
$C_1/C_o = A_1/A_o$ (24)
$C_2/C_o = -(1 + A_1/A_o)$(25)
Where

$$\eta_1 = Y_o Y_1/Y_2,\ \eta_2 = Y_3 Y_4/Y_5$$
$$Y_o = \tau(F_o R_o - D_o R_o)/(F_2 R_2 - D_2 R_2)$$
$$Y_1 = (C_{13} - C_{33})(D_2\mu_2 - F_2 R_2)\sin\theta_2\cos\theta_2 + e_{33}(E_2\mu_2 - F_2\chi_2)\cos\theta_2 + \gamma_3(D_2\chi_2 - E_2 R_2)/ik_2$$
$$Y_2 = (C_{13}\sin^2\theta_2 + C_{33}\cos^2\theta_o)(D_o\mu_o - F_o R_o) - e_{33}(E_o\mu_o - F_o\chi_o)\cos\theta_o + \gamma_3(D_o\chi_o - E_o R_o)/ik_o$$
$$Y_3 = \tau(D_o\mu_o - F_o R_o)/(D_2\mu_2 - F_2 R_2)$$
$$Y_4 = [C_{44}(D_2\mu_2 - F_2 R_2)\cos 2\theta_2 + e_{15}(E_2\mu_2 - F_2\chi_2)\sin\theta_2]$$
$$Y_5 = [C_{44}(D_o\mu_o - F_o R_o)\sin 2\theta_o - e_{15}(E_o\mu_o - F_o\chi_o)\sin\theta_o]$$

We obtain the amplitude ratios from the equations (20-25).

V. APPLICATION TO PARTICULAR MODEL

The reflection coefficients of reflected *QP*, *QS*, *T* and φ waves depend upon angle of incidence, angle of reflection, various elastic and thermopiezoelectric parameters, initial stress and relaxation times. The effect of these parameters on the reflection coefficients may be analyzed for particular model of the medium. For the purpose of numerical computations, the following physical constants of Cadmium Selenide (CdSe) for lower medium are considered [9]

$$C_{11} = 7.41\times10^{10}\ \mathrm{Nm^{-2}},\ C_{12} = 4.52\times10^{10}\ \mathrm{Nm^{-2}},$$
$$C_{13} = 3.93\times10^{10}\ \mathrm{Nm^{-2}},\ C_{33} = 8.36\times10^{10}\ \mathrm{Nm^{-2}},$$
$$C_{44} = 1.32\times10^{10}\ \mathrm{Nm^{-2}},\ \rho = 5504\ \mathrm{Kgm^{-3}},$$
$$e_{13} = -0.160\ \mathrm{Cm^{-2}},\ e_{33} = 0.347\ \mathrm{Cm^{-2}},$$
$$e_{15} = -0.138\ \mathrm{Cm^{-2}},\ T_o = 298\ \mathrm{K},$$
$$\gamma_1 = 0.621\times10^{6}\mathrm{NK^{-1}m^{-2}},\ \gamma_3 = 0.551\times10^{6}\mathrm{NK^{-1}m^{-2}},$$
$$d_3 = -2.94\times10^{-6}\mathrm{CK^{-1}m^{-2}},\ K_1 = K_3 = 9\ \mathrm{Wm^{-1}K^{-1}}$$
$$\epsilon_{11} = 8.26\times10^{-11}\mathrm{C^2N^{-1}m^{-2}},$$
$$\epsilon_{33} = 9.03\times10^{-11}\mathrm{C^2N^{-1}m^{-2}}$$
$$C_E = 260\ \mathrm{J\,Kg^{-1}K^{-1}},\ \omega = 2.14\times10^{13}\mathrm{s^{-1}},$$
$$t_o = 10^{-12} = 1\ \text{pico} - \text{sec}.$$

The variations of phase velocities computed from

$$c_{Lo} = c_{L1} = \sqrt{C_{44} + C_{11}\sin^2\alpha + C_{33}\cos^2\alpha + v_1}\Big/\sqrt{2\rho},$$
$$c_{T2} = \sqrt{C_{44} + C_{11}\sin^2\alpha + C_{33}\cos^2\alpha - v_1}\Big/\sqrt{2\rho},$$

where $v_1 = \sqrt{v_{11} + v_{12}}$,

$$v_{11} = [(C_{11} - C_{44})\sin^2\alpha + (C_{44} - C_{33})\cos^2\alpha]^2,$$
$$v_{12} = (C_{13} + C_{44})^2\sin^2 2\alpha$$

Numerical computations are restricted to incident *QP* wave only. For the incidence of *QP* wave, the reflection coefficients of *QP*, *QS* and *T* waves are computed for Lord and Shulman (L-S) theory with the angle of incidence after using the above physical constants. For L-S theory, the reflection coefficients are computed for the range $0 \le \theta_o \le 90^o$ of angle of incidence, and plotted in figures 2-9 which have the following observations:

Figures2,3,4and 5 represent the relation between the imaginary part of reflection coefficient A_1/A_o, A_2/A_o, B_2/B_o and C_2/C_o as function of incident angle θ_o for various value of relaxation time for (L-S) model. While **Figures** 6,7,8 and 9show the effect of initial stress on the imaginary part of reflection coefficient A_1/A_o, A_2/A_o, B_2/B_o and C_2/C_o as function of incident angle θ_o for (L-S) model.

The real parts of those reflection coefficients have not any influence by the change of the thermal relaxation time in the model of (Lord-Shulman). While the real parts of those reflection coefficients have some affected by the change of the initial stress (we did not give the figures and the details for these influence due to a shortcut the contribution and the details will be given in the coming search).

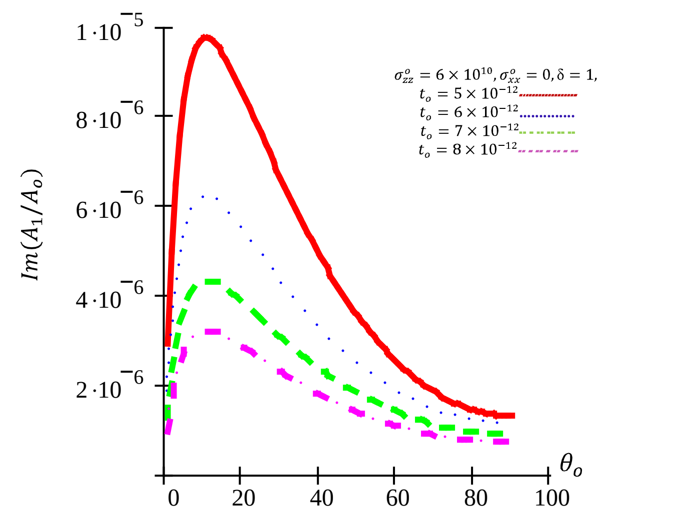

Fig. 2 Imaginary part of reflection coefficientA_1/A_o,as a function ofincidence angle $\theta_o$for various values of the relaxation times

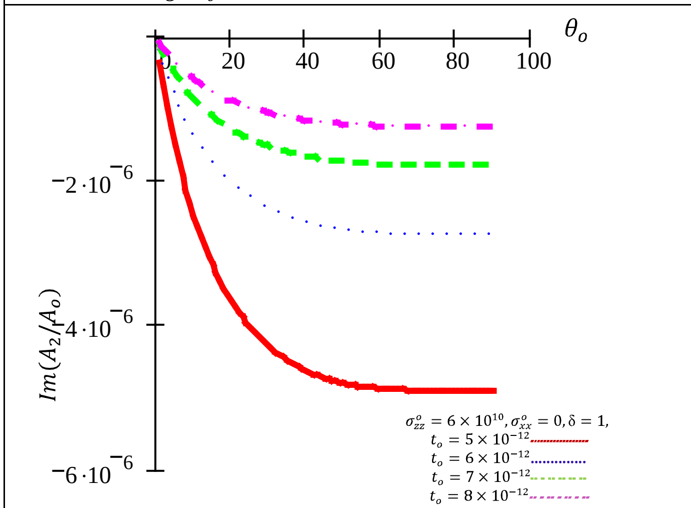

Fig. 3 Imaginary part of reflection coefficient A_2/A_oas a function of incidence angle θ_o for various values of the relaxation times.

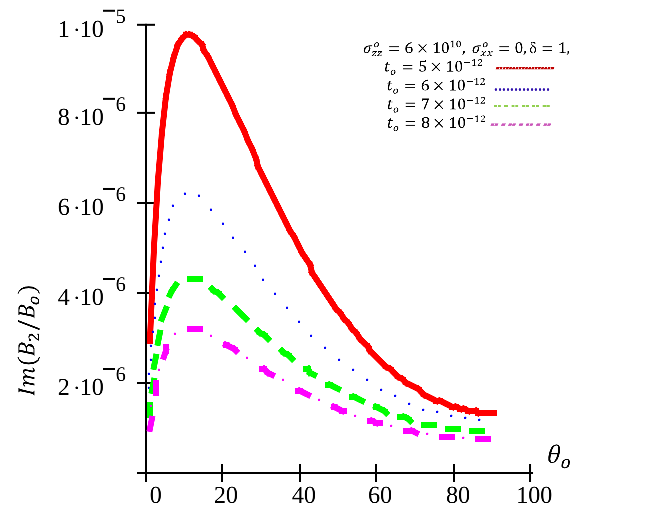

Fig. 4 Imaginary part of reflection coefficient (B_2/B_o) as a function of incidence angle θ_o for various values of the relaxation times.

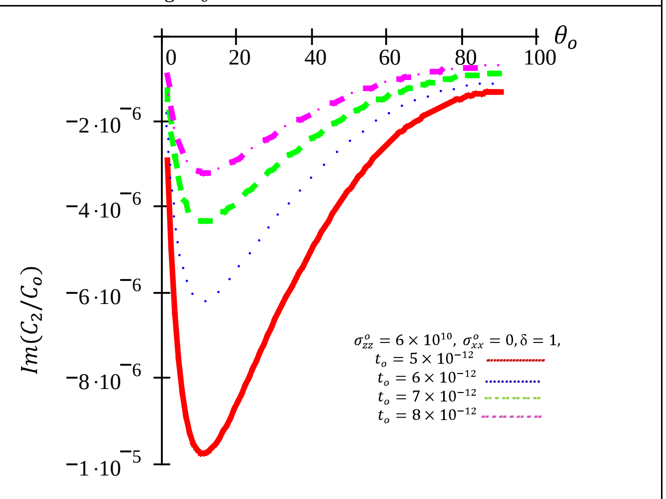

Fig. 5 Imaginary part of reflection coefficient (C_2/C_o) as a function of incidence angle θ_o for various values of the relaxation times.

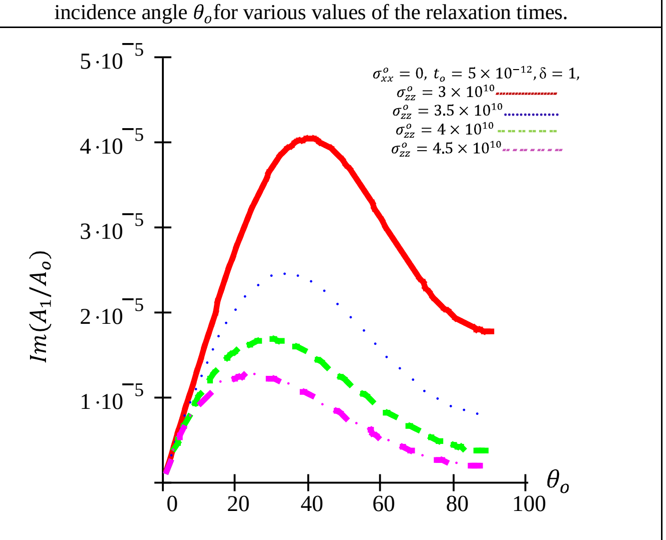

Fig. 6 Imaginary part of reflection coefficient (A_1/A_o) as a function of incidence angle θ_o under effect of different values of the initial stress

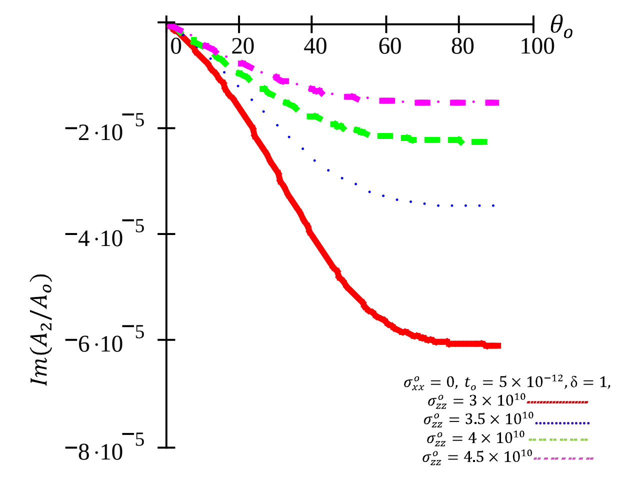

Fig. 7 Imaginary part of reflection coefficient A_2/A_o as a function of incidence angle θ_o under effect of different values of the initial stress.

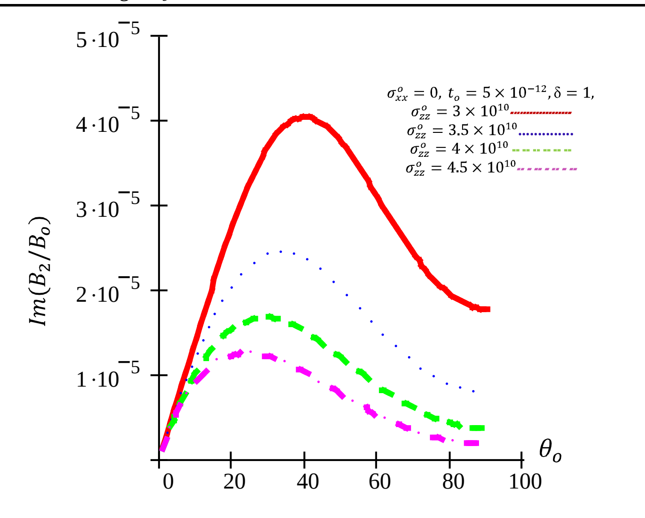

Fig. 8. Imaginary part of reflection coefficient (B_2/B_o) as a function of incidence angle θ_o under effect of different values of the initial stress

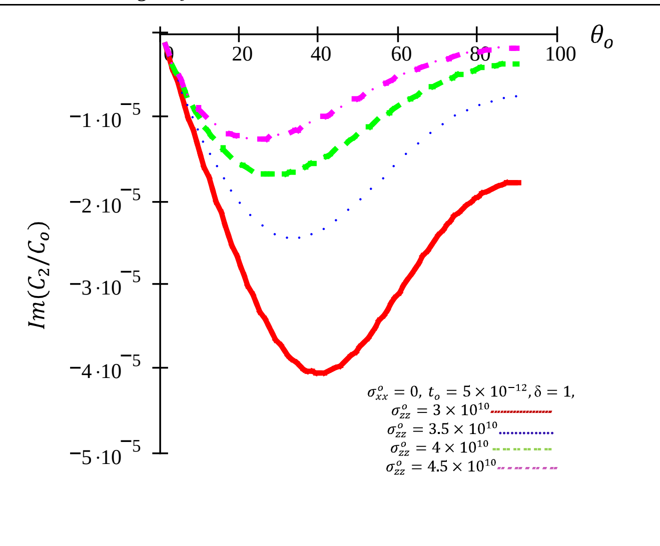

Fig. 9. Imaginary part of reflection coefficient (C_2/C_o) as a function of incidence angle θ_o under effect of different values of the initial stress.

VI. CONCLUSION

Reflection of the plane waves is studied under initial stress of hexagonal thermopiezoelectric solid half-space. The reflection coefficients which dependon the angle of incidence, angle of reflection, various elastic and thermopiezoelectric parameters are computed for a particular model. From numerical computations, it is noticed that the reflection coefficients of various quasi-plane waves are affected significantly due to the presence of relaxation time and anisotropy in the medium. In particular, the *T* wave is most affected due to the presence of relaxation time and anisotropy. However, the reflection coefficients of *T* wave are much less than that of *QP* at each angle of incidence.

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