

# Predator-Prey Model for Noctiluca Scintillans sp. and Acartia sp.

Apisit Jitman, Natdanai Chailangka, Sineenart Srimongkol\*  
Department of Mathematics, Faculty of Science, Burapha University,  
Mueng, Chonburi, 20131  
Thailand

\*Corresponding author, e-mail:sineenart@buu.ac.th

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**Abstract-** In this paper, we have an intention to study about application of mathematical modeling Predator-Prey model to Noctiluca scintillans which is the principal to occur plankton bloom phenomenon in Bangsaen, Thailand where the authors live. We examine Noctiluca scintillans in a prey role meanwhile Arcatia sp. is chosen to be a predator role and observe the results by Lotka Voltera Predator-Prey model though numerical calculation using Python programming.

**Keywords-** Noctiluca Scintillans, Acartia, predatory-prey model, plankton bloom

## I. INTRODUCTION

THE huge accumulation of phytoplankton, macroalgae and colorless heterotrophic protists can change the color of water, cover the surface, make toxicity to ecology ([14]). In the gulf of Thailand, the *N. scintillans* is the most popular phytoplankton such that accumulate and bloom. Since the authors live in Bangsaen district of Chonburi which is part of gulf of Thailand, we face the plankton bloom phenomenon almost every year. The sea is filled by accumulated *N. scintillans*, so it incur toxicity which is globally known as dead sea. Dead creatures of the sea lay whole the beach. Furthermore, it also produce air pollution which annoy the tourists. We have an intention to apply mathematical model to study incidence of this phenomenon in predator and prey role. Moreover, the principal of this phenomenon is *N. scintillans*. Therefore, we are going to add it to this work.

*N. scintillans* or Noctiluca scintillans was classified into the jellyfish until Haeckel decided to move it to the cystoflagellates with the dinoflagellates in 1873. *N. scintillans* is a single-celled spheroid organism and its ranging from is around 400 to 1500  $\mu m$  in length. It can not really swim but can move with the current. *N. scintillans* can exist in a green or red form only, depending on the pigmentation in its vacuoles. It can be found worldwide. Precisely, *N. scintillans* has a significant place in

the pelagic food chain. *N. scintillans* is preyed upon by many copepods such as *Calanus* sp., *Temora* sp. and *Acartia* sp. Because of its excessive proliferation, it always attract many predators due to its very dense aggregations and it frequent bioluminescence in phase of its life.

*Acartia* is known as a genus of marine calanoid copepods. They are epipelagic, estuarine, zooplanktonic found throughout the oceans of the world, primarily in temperate regions. When *Acartia* was just egg or larva, it was preyed by *N. scintillans*. But if it grow up, the role play is switched [1].

In 1968-1988, G.Uhlig and G.Sahling [13] studied about seasonal development of *N. scintillans* with continuous monitoring at a Helgoland-Roads over a period of 20 years. They revealed that the spatial distribution of Noctiluca as determined every year in July, serves as a realistic base for biomass calculations.

In 1998, Y. Nakamura [9] has published Growth and grazing of a large of heterotrophic dinoflagellate, Noctiluca scintillans, in laboratory cultures which it is studying about examining the growth of *N. scintillans* in his laboratory.

In 2003-2004, T. Lirdwitayaprasit, S. Meksumpun, S. Rungsupa and K. Furuya [8] studied the seasonal variations in cell abundance of *N. scintillans* though eight station of the coastal waters of Chonburi Province, the upper gulf of Thailand.

In 2015, M. Freilich [2] used non-dimensional parameters to develop general principal of bloom dynamic with limited light that could test against empirical results.

In 2021, Zhang, Shuwen and Xia, Xiaomin and Ke, Ying and Song, Shuqun and Shen, Zhuo and Cheung, Shunyan and Liu, Hongbin [15] had long survey to conduct in Port Shelter, a subtropical coastal water in NW Pacific, beginning before Noctiluca scintillans and Mesodinium rubrum blooms.

Recently, the survey research in English showed that harmful algal bloom on shellfish, the toxins in mussel can have severe ecological, human health, and seafood industry [10]. Therefore, the study of plankton is crucial.

## II. MATHEMATICAL MODEL

### A. Logistic Model

The nonlinear equation

$$P_{n+1} = rP_n \left(1 - \frac{P_n}{C}\right) \quad (1)$$

where  $r \geq 0$  and  $C > 0$ , called the logistic equation, is a density-dependent model for the population  $P_n$  of a species after  $n$  generations [11].

### B. Predator-Prey Model

Volterra wrote down the following model:

$$\frac{dx}{dt} = ax - bxy, \quad (2)$$

$$\frac{dy}{dt} = -cy + dxy, \quad (3)$$

where  $x$  and  $y$  represent prey and predator populations respectively,  $t$  is the time, and note that  $a$  is the rate of birth of prey population,  $b$  is the rate of death of prey population,  $c$  is the rate of death of predator population and  $d$  is rate of birth of predator population [12].

The matrix notation of the system of equations (2) and (3) is shown in the following equation,

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}, \quad (4)$$

where,  $\mathbf{x}$  and  $\mathbf{F}$  are matrix of the form

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} f_1(x, y) \\ f_2(x, y) \end{bmatrix} = \begin{bmatrix} ax - bxy \\ -cy + dxy \end{bmatrix}.$$

Then we will consider an eco-epidemiological system consisting of two types of species. The prey population is Noctiluca and the predator population is Acartia. For simplicity, we assume that the prey population grows logistically and the predator populations grow exponentially. Then the above model is changed as the following:

$$\frac{dx}{dt} = \alpha x \left(1 - \frac{x}{C}\right) - \beta xy, \quad (5)$$

$$\frac{dy}{dt} = -\delta y + \gamma xy, \quad (6)$$

Note that

- $\alpha$  is the rate of birth of prey population (Noctiluca).
- $\beta$  is the rate of death of prey population (Noctiluca).
- $\delta$  is the rate of death of predator population (Acartia).
- $\gamma$  is the rate of birth of predator population (Acartia).
- $C$  is the carrying capacity of prey population (Noctiluca).

In 1997, the growth rate of N.scintillans was examined in Nakamura's laboratory [9], so  $\alpha$  is 0.2 cell/t corresponding with the study in Australia [4]. In 2009, P. Jitchum and L. Wongrat [3] studied the community structure and abundance of epipelagic copepods in a shallow protected bay, Gulf of Thailand, therefore, we obtain the rate of birth of Acartia sp. which is part of copepods such that its rate is  $\delta = 0.1$  cell/t. In fact, we

Table 1: Parameters using in the simulation.

parameter	value
$\alpha$	0.2 cell/t
$\beta$	0.1 cell/t
$\delta$	0.4 cell/t
$\gamma$	0.1 cell/t
$C$	100 cell

do not have information for the others, we will assume parameter rate of death of prey population and rate of death of predator population as follows  $\beta$  and  $\gamma$  are 0.1 and 0.4 cell/t respectively. And the last one the carrying capacity is 100 cell.

Thus, we will establish the model by satisfying the assumptions as the following:

1. The Acartia sp. which is predator must be adult state.
2. Both of N. scintillans and Acartia sp. are not allowed to migrate across territory of 1 lit of water.

## III. QUALITATIVE ANALYSIS

To analyse the behavior of the system of equation (5) and (6), the steady state solutions and Jacobian Matrix are calculated. From equations (5) and (6), the steady state solutions is obtained by making the change to zero as shown in the following equations,

$$\alpha x \left(1 - \frac{x}{C}\right) - \beta xy = 0, \quad (7)$$

$$-\delta y + \gamma xy = 0, \quad (8)$$

Solve the system of equations (7) and (8), the three steady states,  $(\bar{x}_1, \bar{y}_1)$ ,  $(\bar{x}_2, \bar{y}_2)$ ,  $(\bar{x}_3, \bar{y}_3)$ , are as follow:

$$\bar{x}_1 = 0, \quad \bar{y}_1 = 0 \quad (9)$$

$$\bar{x}_2 = C, \quad \bar{y}_2 = 0 \quad (10)$$

$$\bar{x}_3 = \frac{\delta}{\gamma}, \quad \bar{y}_3 = \frac{\alpha}{\beta} \left(1 - \frac{\delta}{\gamma C}\right). \quad (11)$$

To make the equations (5) and (6) linear, the Jacobian Matrix is calculated by its definition as shown in the equation (12).

$$J = \begin{bmatrix} \alpha - \frac{2\alpha x}{C} - \beta y & -\beta x \\ \gamma y & -\delta + \gamma x \end{bmatrix} \quad (12)$$

The stability is analyzed at each steady states. For  $(0, 0)$ , we obtain

$$J(0, 0) = \begin{bmatrix} \alpha & 0 \\ 0 & -\delta \end{bmatrix}. \quad (13)$$

Thus

$$\text{Tr}(J) = \alpha - \delta = -0.2, \quad (14)$$

$$\det(J) = -\alpha\delta = -0.08, \quad (15)$$

$$\text{disc}(J) = (\alpha - \delta)^2 + 4(\alpha\delta) = 0.36, \quad (16)$$

namely ,  $\text{Tr}(J) < 0$  meanwhile  $\det(j) < 0$ . Therefore,  $(0, 0)$  is a saddle point.

For  $(C, 0)$ , we obtain

$$J(C, 0) = \begin{bmatrix} -\alpha & -\beta C \\ 0 & -\delta + \gamma C \end{bmatrix}. \quad (17)$$

Thus

$$\text{Tr}(J) = -\alpha - \delta + \gamma C = 9.4, \quad (18)$$

$$\det(J) = -\alpha(\delta + \gamma C) = -2.08, \quad (19)$$

$$\text{disc}(J) = (-\alpha - \delta + \gamma C)^2 + 4\alpha(\delta + \gamma C) = 96.68, \quad (20)$$

namely ,  $\text{Tr}(J) > 0$  meanwhile  $\det(j) < 0$ . Therefore,  $(C, 0)$  is a saddle point.

For  $(\frac{\delta}{\gamma}, \frac{\alpha}{\beta} (1 - \frac{\delta}{\gamma C}))$ , we obtain

$$J(\frac{\delta}{\gamma}, \frac{\alpha}{\beta} (1 - \frac{\delta}{\gamma C})) = \begin{bmatrix} -\frac{\alpha\delta}{\gamma C} & -\frac{\beta\delta}{\gamma} \\ \frac{\alpha}{\beta} (\gamma - \frac{\delta}{C}) & 0 \end{bmatrix}. \quad (21)$$

Thus

$$\text{Tr}(J) = -\frac{\alpha\delta}{\gamma C} = -0.008, \quad (22)$$

$$\det(J) = \frac{\alpha\delta}{\gamma} (\gamma - \frac{\delta}{C}) = 0.0768, \quad (23)$$

$$\text{disc}(J) = (-\frac{\alpha\delta}{\gamma C})^2 - \frac{4\alpha\delta}{\gamma} (\gamma - \frac{\delta}{C}) = -0.076736, \quad (24)$$

namely ,  $\text{Tr}(J) < 0$  meanwhile  $\det(j) > 0$ . Therefore,  $(\frac{\delta}{\gamma}, \frac{\alpha}{\beta} (1 - \frac{\delta}{\gamma C}))$  is a stable spiral.

#### IV. NUMERICAL INVESTIGATION

In this section, the model in equations 5 and 6 is used predict the relation between Noctiluca and Acartia through the numerical result by Python programming. With the initial values 10 N. scintillans  $cells/cm^3$  and 1 Acartia  $cells/cm^3$ . The results is shown in Fig. 1. The N. scintillans and Acartia are oscilate when N. scintillans grows fast the Acartia grow slow and vice versa. The periodic solution is suit to model the behavior of the N. scintillans and Acartia.

The comparison of the behavior of the N. scintillans from numerical result and investigated information by Likhit, Chuchit and Chalemchia, Yoosamran ([5], [6], [7]) is presented in Fig. 2.

From figure 1 and figure 2, the trend of Predator-Prey model's graph analogous to real information even the result from model is lower than the real value.

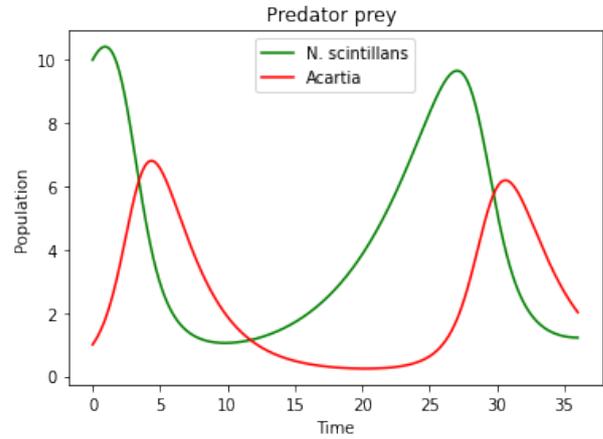


Fig. 1: Predator-Prey model predict dynamic between population of Noctiluca and Acartia ( $cell/cm^3$ ) where the time unit is month.

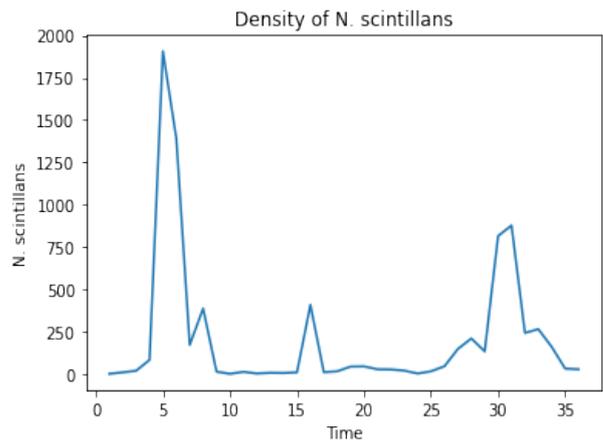


Fig. 2: Information of N. scintillans ( $cell/cm^3$ ) with continued observing 3 years.

## V. CONCLUSION

The result which is obtained from Predator-Prey model correlate highly with real investigated information. We will see the intersection between *N. scintillans* and *Acartia* sp. in Figure 1 such that there will be turning point after that. It means that if *N. scintillans* is the prey and decreased into that point there will be changing to increasing rate of Predator. Then the increasing population of *Acartia* sp. will face the problem to consume imply the *Acartia* sp.'s population decrease. In technically, at the second intersection point there will arise new turning point making the *N. scintillans*'s population highly increase. If we observe in a gap of time, this phenomenon will occur in seasoning every year but before the worst case effect from plankton bloom happen, the predator population will significantly increase and stop it promptly.

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