

Mathematical modeling of large forest fires initiation and spread

V.A. Perminov

Tomsk Polytechnic University, Tomsk, Russia

Abstract—It is presented mathematical model for description of heat and mass transfer processes at crown and large forest fires initiation and spread, taking into account their mutual influences. Mathematical model of forest fire was based on an analysis of experimental data and using concepts and methods from reactive media mechanics. The paper suggested in the context of the general mathematical model of forest fires give a new mathematical setting and method of numerical solution of a problem of a forest fire modeling. The boundary-value problem is solved numerically using the control volume methods and method of splitting according to physical processes. In this context, a study - mathematical modeling - of the conditions of forest fire initiation and spreading that would make it possible to obtain a detailed picture of the change in the velocity, temperature and component concentration fields with time, and determine as well as the conditions of forest fires initiation and spread is of interest.

Keywords— Mathematical model, forest fire, turbulence, combustion, control volume, discrete analogue.

I. INTRODUCTION

One of the objectives of these studies is the improvement of knowledge on the fundamental physical mechanisms that control forest fires initiation and spread. A great deal of work has been done on the theoretical problem of how forest fire initiation. In forest there are two steps for crown forest fire initiation: spread of fire from crow to crown and crown fires are initiated by convective and radiative heat transfer from surface fires. However, convection is the main heat transfer mechanism. Firstly crown forest fire initiation have been studied and modeled by Van Wagner [1]. There are three simple crown properties: crown base height, bulk density and moisture content of forest fuel in this theory. Also crown fire initiation have been studied and modeled in detail (eg: Alexander [2], Van Wagner [3], Xanthopoulos, [4], Rothermel [5,6], Van Wagner, [7], Cruz [8], Albini [9], Scott, J. H. and Reinhardt, E. D. [10].The discussion of the problem of modeling forest fires is provided by a group of co-workers at Tomsk University (Grishin [11], Grishin and Perminov [12], Perminov [13,14]). The general

mathematical model of forest fires was obtained by Grishin [11] based on an analysis of known and original experimental data [11,15], and using concepts and methods from reactive media mechanics. The physical two-phase models used in [16-17] may be considered as a continuation and extension of the formulation proposed by Grishin and Perminov [12-14]. However, the investigation of crown fires has been limited mainly to cases studied of forest fires initiation in two dimensional settings and did not take into account space properties of these phenomena. One of the objectives of these studies is the improvement of knowledge on the fundamental physical mechanisms that control forest fires initiation and spread. A great deal of work has been done on the theoretical problem of how forest fire initiation. In forest there are two steps for crown forest fire initiation: spread of fire from crow to crown and crown fires are initiated by convective and radiative heat transfer from surface fires. However, convection is the main heat transfer mechanism. Firstly crown forest fire initiation have been studied and modeled by Van Wagner [1]. There are three simple crown properties: crown base height, bulk density and moisture content of forest fuel in this theory. Also crown fire initiation have been studied and modeled in detail (eg: Alexander [2], Van Wagner [3], Xanthopoulos, [4], Rothermel [5,6], Van Wagner, [7], Cruz [8], Albini [9], Scott, J. H. and Reinhardt, E. D. [10].The discussion of the problem of modeling forest fires is provided by a group of co-workers at Tomsk University (Grishin [11], Grishin and Perminov [12], Perminov [13,14]). The general mathematical model of forest fires was obtained by Grishin [11] based on an analysis of known and original experimental data [11,15], and using concepts and methods from reactive media mechanics. The physical two-phase models used in [16-17] may be considered as a continuation and extension of the formulation proposed by Grishin and Perminov [12-14]. However, the investigation of crown fires has been limited mainly to cases studied of forest fires initiation in two dimensional settings and did not take into account space properties of these phenomena.

II. PHYSICAL AND MATHEMATICAL MODEL

It is assumed that the forest during a forest fire can be modeled as 1) a multi-phase, multistoried, spatially heterogeneous medium; 2) in the fire zone the forest is a porous-dispersed, two-temperature, single-velocity, reactive medium; 3) the forest canopy is supposed to be non-deformed medium (trunks, large branches, small twigs and needles), affects only the magnitude of the force of resistance in the equation of conservation of momentum in the gas phase, i.e., the medium is assumed to be quasi-solid (almost non-deformable during wind gusts); 4) let there be a so-called “ventilated” forest massif, in which the volume of fractions of condensed forest fuel phases, consisting of dry organic matter, water in liquid state, solid pyrolysis products, and ash, can be neglected compared to the volume fraction of gas phase (components of air and gaseous pyrolysis products); 5) the flow has a developed turbulent nature and molecular transfer is neglected; 6) gaseous phase density doesn't depend on the pressure because of the low velocities of the flow in comparison with the velocity of the sound. Let the coordinate reference point $x_1, x_2, x_3 = 0$ be situated at the centre of the domain of surface forest fire source at the height of the roughness level, axis $0x_1$ directed parallel to the horizontal surface to the right in the direction of the unperturbed wind speed, axis $0x_2$ directed perpendicular to $0x_1$ and axis $0x_3$ directed upward (Fig. 1).

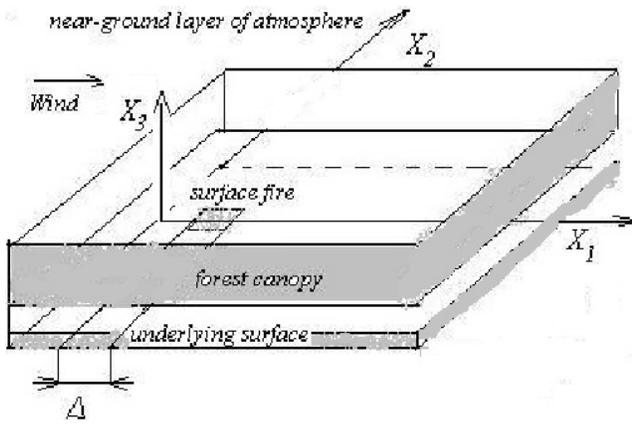


Fig. 1 Schematic of a forest fire domain.

Using the results of [11-14] and known experimental data [15] we have the following sufficiently general equations, which define the state of the medium in the forest fire zone, written using tensor notation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho v_j) = \dot{m}, \quad j = 1, 2, 3, \quad i = 1, 2, 3; \quad (1)$$

$$\rho \frac{dv_i}{dt} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} (-\rho \overline{v'_i v'_j}) - \rho \sigma c_d v_i |\vec{v}| - \rho g_i - \dot{m} v_i; \quad (2)$$

$$\rho c_p \frac{dT}{dt} = \frac{\partial}{\partial x_j} (-\rho c_p \overline{v'_j T'}) + q_3 R_5 - \alpha_v (T - T_s) + k_g (c U_R - 4 \sigma T^4) \quad (3)$$

$$\rho \frac{dc_\alpha}{dt} = \frac{\partial}{\partial x_j} (-\rho \overline{v'_j c'_\alpha}) + R_{5\alpha} - \dot{m} c_\alpha, \quad \alpha = \overline{1, 5}; \quad (4)$$

$$\frac{\partial}{\partial x_j} \left(\frac{c}{3k} \frac{\partial U_R}{\partial x_j} \right) - kc U_R + 4k_s \sigma T_s^4 + 4k_g \sigma T^4 = 0, \quad (5)$$

$$k = k_g + k_s;$$

$$\sum_{i=1}^4 \rho_i c_{pi} \varphi_i \frac{\partial T_s}{\partial t} = k_s (c U_R - 4 \sigma T_s^4) + \quad (6)$$

$$+ q_3 R_3 - q_2 R_2 + \alpha_v (T - T_s);$$

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = -R_1, \quad \rho_2 \frac{\partial \varphi_2}{\partial t} = -R_2, \quad (7)$$

$$\rho_3 \frac{\partial \varphi_3}{\partial t} = \alpha_c R_1 - \frac{M_c}{M_1} R_3, \quad \rho_4 \frac{\partial \varphi_4}{\partial t} = 0;$$

$$\sum_{\alpha=1}^5 c_\alpha = 1, \quad p_e = \rho RT \sum_{\alpha=1}^5 \frac{c_\alpha}{M_\alpha}, \quad \vec{v} = (v_1, v_2, v_3), \quad \vec{g} = (0, 0, g)$$

$$\dot{m} = (1 - \alpha_c) R_1 + R_2 + \frac{M_c}{M_1} R_3 + R_{54} + R_{55},$$

$$R_{51} = -R_3 - \frac{M_1}{2M_2} R_5, \quad R_{52} = v_g (1 - \alpha_c) R_1 - R_5,$$

$$R_{53} = 0, \quad R_{54} = \alpha_4 R_1, \quad R_{55} = \frac{\alpha_5 v_3}{v_3 + v_3^*} R_3.$$

$$R_1 = k_1 \rho_1 \varphi_1 \exp\left(-\frac{E_1}{RT_s}\right), \quad R_2 = k_2 \rho_2 \varphi_2 T^{-0.5} \exp\left(-\frac{E_2}{RT_s}\right),$$

$$R_3 = k_3 \rho \varphi_3 S_\sigma c_1 \exp\left(-\frac{E_3}{RT_s}\right),$$

$$R_5 = k_5 M_2 \left(\frac{c_1 M}{M_1} \right)^{0.5} \left(\frac{c_2 M}{M_2} \right) T^{-2.25} \exp\left(-\frac{E_5}{RT}\right).$$

Here and above $\frac{d}{dt}$ is the symbol of the total (substantial) derivative. The system of equations (1)–(7) must be solved taking into account the initial and boundary conditions:

$$x_1 = -x_{1e} : v_1 = V_e, \quad v_2 = 0, \quad \frac{\partial v_3}{\partial x_1} = 0, \quad T = T_e, \quad c_\alpha = c_{ae}, \quad (8)$$

$$-\frac{c}{3k} \frac{\partial U_R}{\partial x_1} + c U_R / 2 = 0;$$

$$x_1 = x_{1e} : \frac{\partial v_1}{\partial x_1} = 0, \frac{\partial v_2}{\partial x_1} = 0, \frac{\partial v_3}{\partial x_1} = 0, \frac{\partial c_\alpha}{\partial x_1} = 0, \quad (9)$$

$$\frac{\partial T}{\partial x_1} = 0, \frac{c}{3k} \frac{\partial U_R}{\partial x_1} + \frac{c}{2} U_R = 0 ;$$

$$x_2 = x_{20} : \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \quad (10)$$

$$\frac{\partial T}{\partial x_2} = 0, -\frac{c}{3k} \frac{\partial U_R}{\partial x_2} + \frac{c}{2} U_R = 0 ;$$

$$x_2 = x_{2e} : \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \quad (11)$$

$$\frac{\partial T}{\partial x_2} = 0, \frac{c}{3k} \frac{\partial U_R}{\partial x_2} + \frac{c}{2} U_R = 0 .$$

$$x_3 = 0 : v_1 = 0, v_2 = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, -\frac{c}{3k} \frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0, \quad (12)$$

$$v_3 = v_{30}, T = T_g, |x_1| \leq \Delta, |x_2| \leq \Delta$$

$$v_3 = 0, T = T_e, |x_1| > \Delta, |x_2| > \Delta ;$$

$$x_3 = x_{3e} : \frac{\partial v_1}{\partial x_3} = 0, \frac{\partial v_2}{\partial x_3} = 0, \frac{\partial v_3}{\partial x_3} = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, \quad (13)$$

$$\frac{\partial T}{\partial x_3} = 0, \frac{c}{3k} \frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0 .$$

Here α_y is the coefficient of phase exchange; ρ - density of gas – dispersed phase, t is time; v_i - the velocity components; T, T_s - temperatures of gas and solid phases, U_R - density of radiation energy, k - coefficient of radiation attenuation, P - pressure; c_p – constant pressure specific heat of the gas phase, $c_{pi}, \rho_i, \varphi_i$ – specific heat, density and volume of fraction of condensed phase (1 – dry organic substance, 2 – moisture, 3 – condensed pyrolysis products, 4 – mineral part of forest fuel), R_i – the mass rates of chemical reactions, q_i – thermal effects of chemical reactions; k_g, k_s - radiation absorption coefficients for gas and condensed phases; T_e - the ambient temperature; c_α - mass concentrations of α - component of gas - dispersed medium, index $\alpha=1,2,\dots,5$, where 1 corresponds to the density of oxygen, 2 - to gas products of pyrolysis(carbon monoxide CO, CH_4 and etc.), 3 - to carbon dioxide and inert components of air, 4 - to particles of black, 5 - to particles of smoke; R – universal gas constant; M_α, M_C and M molecular mass of α - components of the gas phase, carbon and air mixture; g is the gravity acceleration; c_d is an empirical coefficient of the resistance of the vegetation, s is the specific surface of the forest fuel in the given forest stratum. The initial values for volume of fractions of condensed phases are determined using the expressions:

$$\varphi_{1e} = \frac{d(1-v_z)}{\rho_1}, \varphi_{2e} = \frac{Wd}{\rho_2}, \varphi_{3e} = \frac{\alpha_c \varphi_{1e} \rho_1}{\rho_3}$$

where d -bulk density for surface layer, v_z – coefficient of ashes of forest fuel, W – forest fuel moisture content. It is supposed that the optical properties of a medium are independent of radiation wavelength (the assumption that the medium is “grey”), and the so-called diffusion approximation for radiation flux density were used for a mathematical description of radiation transport during forest fires. To close the system (1)–(7), the components of the tensor of turbulent stresses, and the turbulent heat and mass fluxes are determined using the local-equilibrium model of turbulence (Grishin, [10]). The system of equations (1)–(7) contains terms associated with turbulent diffusion, thermal conduction, and convection, and needs to be closed. The components of the tensor of turbulent stresses $\overline{\rho v_i v_j'}$, as well as the turbulent fluxes of heat and mass $\overline{\rho v_j' c_p T'}$, $\overline{\rho v_j' c_\alpha'}$ are written in terms of the gradients of the average flow properties using the formulas:

$$-\overline{\rho v_i v_j'} = \mu_t \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} K \delta_{ij},$$

$$-\overline{\rho v_j c_p T'} = \lambda_t \frac{\partial T}{\partial x_j}, \quad -\overline{\rho v_j c_\alpha'} = \rho D_t \frac{\partial c_\alpha}{\partial x_j},$$

$$\lambda_t = \mu_t c_p / Pr_t, \quad \rho D_t = \mu_t / Sc_t, \quad \mu_t = c_\mu \rho K^2 / \varepsilon,$$

where μ_t, λ_t, D_t are the coefficients of turbulent viscosity, thermal conductivity, and diffusion, respectively; Pr_t, Sc_t are the turbulent Prandtl and Schmidt numbers, which were assumed to be equal to 1. In dimensional form, the coefficient of dynamic turbulent viscosity is determined using local equilibrium model of turbulence [11]. The length of the mixing path is determined using the formula $l = x_3 k_t / (1 + 2.5 x_3 \sqrt{c_d s / h})$ taking into account the fact that the coefficient of resistance c_d in the space between the ground cover and the forest canopy base is equal to zero, while the constants $k_t = 0.4$ and $h = h_2 - h_1$ (h_2, h_1 – height of the tree crowns and the height of the crown base). It should be noted that this system of equations describes processes of transfer within the entire region of the forest massif, which includes the space between the underlying surface and the base of the forest canopy, the forest canopy and the space above it, while the appropriate components of the data base are used to calculate the specific properties of the various forest strata and the near-ground layer of atmosphere. This approach substantially simplifies the technology of solving problems of predicting the state of the medium in the fire zone numerically. The thermodynamic, thermophysical and structural characteristics correspond to the forest fuels in the canopy of a different (for example pine [10,12]) type of forest.

III. NUMERICAL METHOD AND RESULTS

The boundary-value problem (1)–(13) was solved numerically using the method of splitting according to physical processes [13]. In the first stage, the hydrodynamic pattern of flow and distribution of scalar functions was calculated. The system of ordinary differential equations of chemical kinetics obtained as a result of splitting [13] was then integrated. A discrete analog was obtained by means of the control volume method using the SIMPLE like algorithm [13,18]. The accuracy of the program was checked by the method of inserted analytical solutions. The time step was selected automatically.

Fields of temperature, velocity, component mass fractions, and volume fractions of phases were obtained numerically. Figures 2 illustrates the time dependence of dimensionless temperatures of gas (1) and condensed phases (2), Figure 3 – mass concentrations of gas components (1- oxygen, 2- gas products of pyrolysis), and Figure.4 - relative volume fractions of solid phases (1), moisture (2) and coke (3) at crown base of the forest ($V_e=5\text{m/s}$). At the moment of ignition the gas combustible products of pyrolysis burns away, and the concentration of oxygen is rapidly reduced. The temperatures of both phases reach a maximum value at the point of ignition. The ignition processes is of a gas - phase nature, i.e. initially heating of solid and gaseous phases occurs, moisture is evaporated. Then decomposition process into condensed and volatile pyrolysis products starts, the later being ignited in the forest canopy.

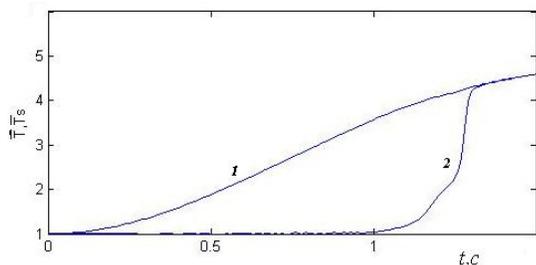


Fig. 2. Temperature of gas (1 - \bar{T}) and solid phase (2 - \bar{T}_S);
 $\bar{T} = T/T_e$, $\bar{T}_S = T_S/T_e$, $T_e = 300\text{K}$.

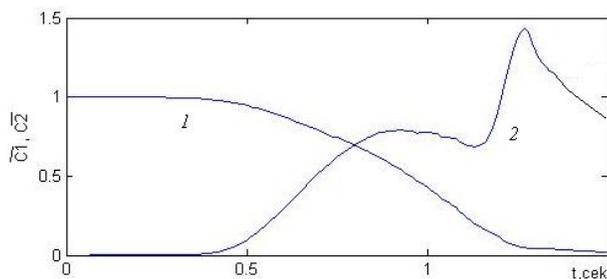


Fig. 3. Concentration of oxygen (1- \bar{c}_1) and gas products of pyrolysis (2- \bar{c}_2); $\bar{c}_\alpha = c_\alpha / c_{1e}$, $c_{1e} = 0.23$.

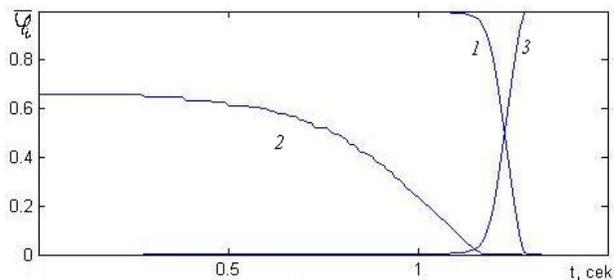


Fig. 4. Volume fractions of solid phase:

$$1 - \bar{\varphi}_1 = \varphi_1 / \varphi_{1e}, 2 - \bar{\varphi}_2 = \varphi_2 / \varphi_{2e}, 3 - \bar{\varphi}_3 = \varphi_3 / \varphi_{3e}.$$

Note also that the transfer of energy from the fire source takes place due to radiation; the value of radiation heat flux density is small compared to that of the convective heat flux. As a result of heating of forest fuel elements, moisture evaporates, and pyrolysis occurs accompanied by the release of gaseous products, which then ignite. At $V_e \neq 0$, the wind field in the forest canopy interacts with the gas-jet obstacle that forms from the forest fire source and from the ignited forest crown and burn away in the forest canopy. Forest fire begins to spread in the forest canopy. The distribution of temperature, concentrations of gas products of pyrolysis and oxygen in the forest fire front are presented in the Figure 5.

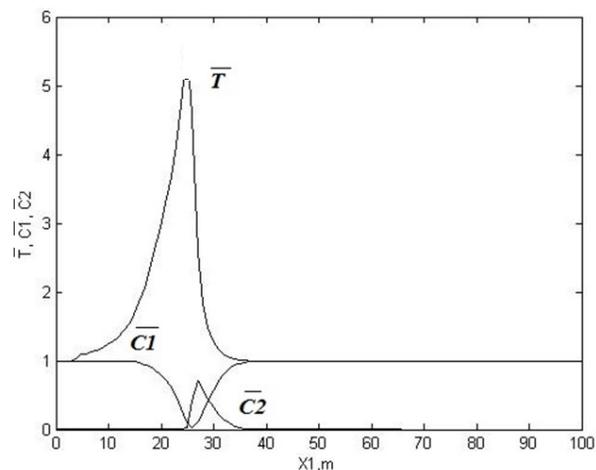


Fig. 5. The distribution of gas phase temperature (\bar{T} ($\bar{T} = T/T_e$, $T_e = 300\text{K}$), concentrations of gas products of pyrolysis \bar{c}_2 ($\bar{c}_2 = c_2 / c_{1e}$, $c_{1e} = 0.23$) and oxygen \bar{c}_1 ($\bar{c}_1 = c_1 / c_{1e}$) in forest fire front.

It is seen that the combustion wave looks like as a soliton. The oxygen concentration drops to near zero in front of a fire. It is consumed in the combustion of the pyrolysis products, the concentration of which is reached their maximum before the maximum of temperature.

Figures 6 – 7 present the distribution of temperature \bar{T} ($\bar{T} = T/T_e$, $T_e = 300\text{K}$) (1- 1.5, 2 - 2., 3 – 2.6, 4 – 3, 5 –

3.5, 6 – 4.) for gas phase, concentrations of oxygen \bar{c}_1 (1 – 0.1, 2 – 0.5, 3 – 0.6, 4 – 0.7, 5 – 0.8, 6 – 0.9) and volatile combustible products of pyrolysis \bar{c}_2 (1 – 1., 2- 0.1, 3 – 0.05, 4 – 0.01) ($\bar{c}_a = c_a / c_{1e}, c_{1e} = 0.23$) at different instants of time for wind velocity $V_e = 5$ m/s (Fig.6) and $V_e = 10$ m/s (Fig.7). The distribution of isotherms of combustion temperature shows the moving of forest fire front with time. Figure 16 shows that with the increase of wind speed up to 10 m/s increases the rate of fire spread to 5 m/sec.

Ignition of forest due to spotting is one of the most difficult aspects to understand the behavior of fires. The phenomenon of spotting fires comprises three sequential mechanisms: generation, transport and ignition of recipient fuel. The present mathematical model and results of calculation are used to illustrate picture of the formation of large fires by combining small combustion sources arising from the transfer of firebrands. In order to understand these mechanisms, many calculation experiments have been performed. In the Figures 8 - 12 the process of formation

large forest fire front as a result of integration of various sources of combustion is showed. The distributions of temperature for gas phase(a), concentrations of oxygen (b) and volatile combustible products of pyrolysis (c) at different times (I - t=3 sec., II - t=6 sec, III - t=12 sec.) for $V_e = 5$ m/s are presented. There are present the same values of isotherms and isolines of concentration for \bar{c}_1 and \bar{c}_2 as well as in the Figures 6-7. If the sources of ignition from the burning particles are arranged in a triangle, the processes of formation of the forest fire front are shown in Figures 8-9. In the second case (Figure 9), the right ignition source in the x_2 direction was 2 times less than in the first case (Figure 8). If all three combustion sources are situated on the same line, the formation of the combustion front is shown in Figure 10. Rather interesting picture of the formation of the front fire is implemented in the case of four combustion sources are located at the corners of a rectangle (Figure 11.) or trapezoid (Figure 12.).

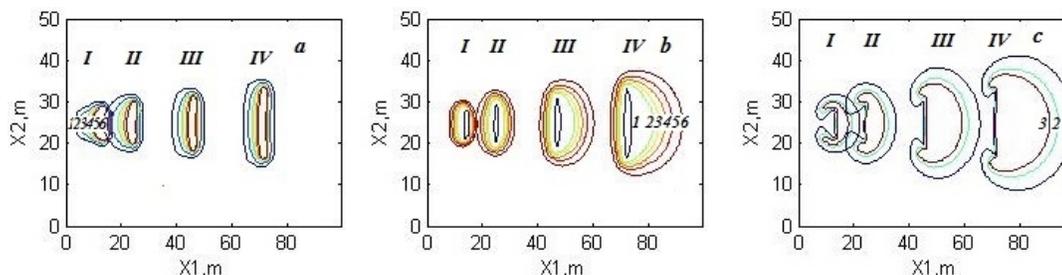


Fig. 6. The distribution of a) temperature for gas phase, b) concentration of oxygen and c) volatile combustible products of pyrolysis; $V_e = 5$ m/s, at different instants of time: I - t=3 sec., II - t=6 sec, III - t=12 sec., IV - t= 20 sec.

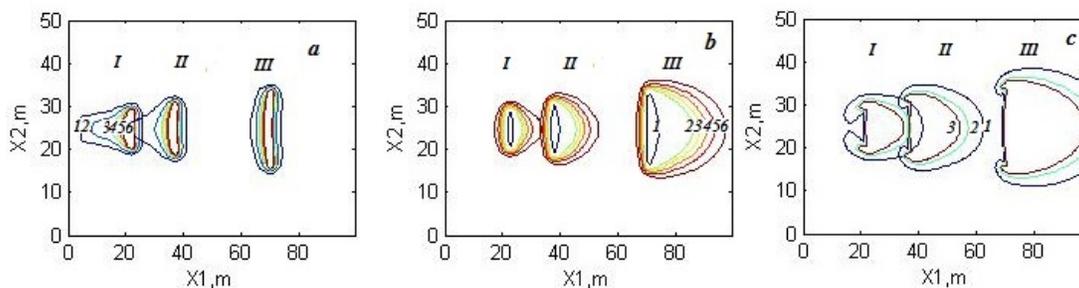


Fig. 7. The distribution of a) temperature for gas phase, b) concentrations of oxygen and c) volatile combustible products of pyrolysis; $V_e = 10$ m/s, at different instants of time: I - t=3 sec., II - t=6 sec, III - t=12 sec., IV - t= 20 sec.

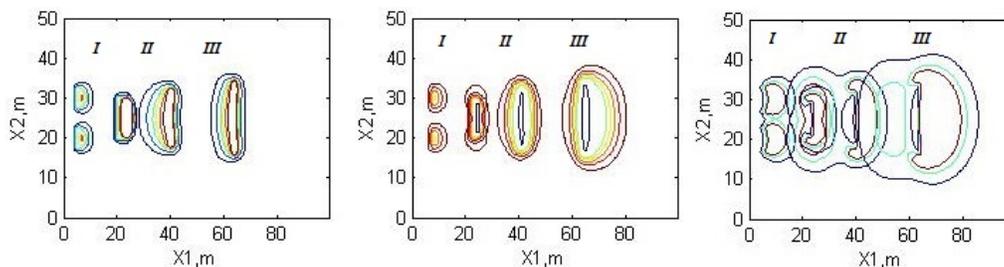


Fig. 8. The distribution of temperature for gas phase, concentrations of oxygen and volatile combustible products of pyrolysis.

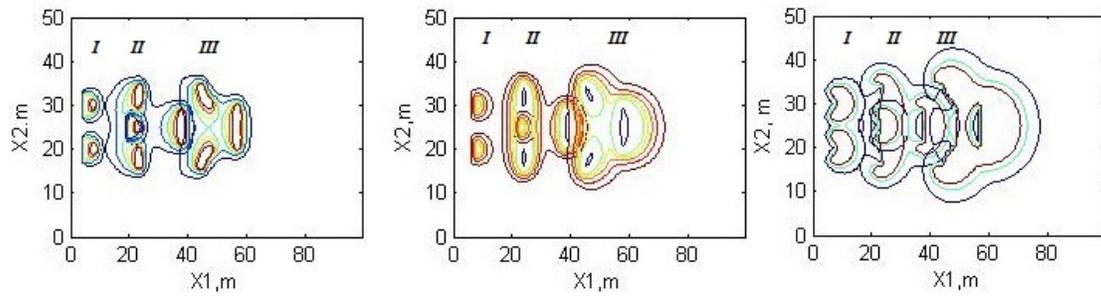


Fig. 9. The distribution of temperature for gas phase, concentrations of oxygen and volatile combustible products of pyrolysis.

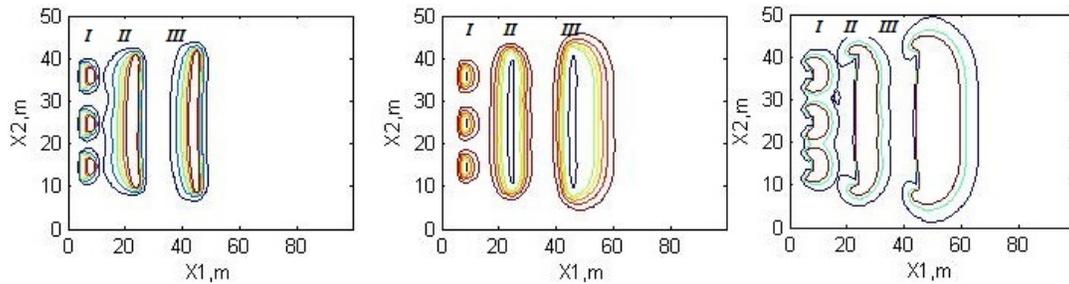


Fig. 10. The distribution of temperature for gas phase, concentrations of oxygen and volatile combustible products of pyrolysis.

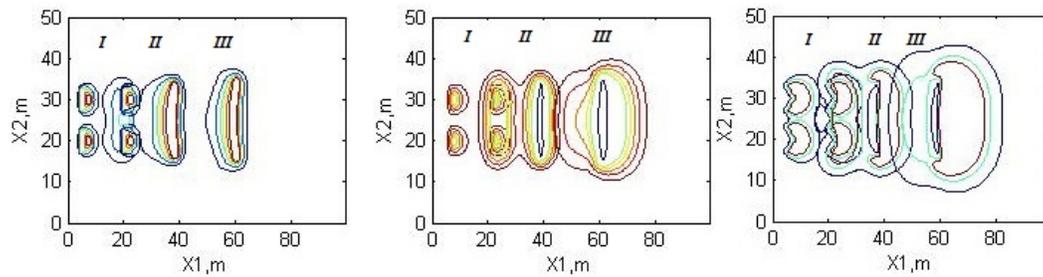


Fig. 11 The distribution of temperature for gas phase, concentrations of oxygen and volatile combustible products of pyrolysis.

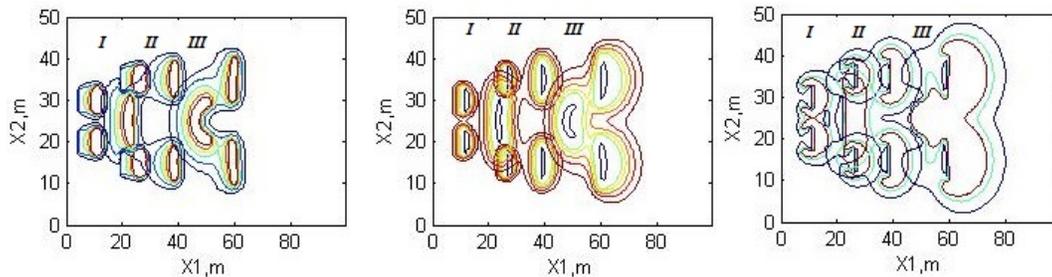


Fig. 12. The distribution of temperature for gas phase, concentrations of oxygen and volatile combustible products of pyrolysis.

IV. CONCLUSION

Using of this model gives an opportunity to describe the different conditions of the large forest fires initiation and spread taking account different weather conditions and state of forest combustible materials, which allows applying the given model for prediction and preventing fires. It overestimates the rate of crown forest fire spread that depends on crown properties: bulk density, moisture content of forest fuel, wind velocity and etc. The model proposed here gives a detailed picture of the change in the temperature and component concentration fields with time, and determine as well as the influence of different conditions on the crown forest fire spreading for the different cases of inhomogeneous of distribution of sources of forest fires initiation. The results of calculation of the rate of crown forest fires are agreed with the laws of physics and experimental data.

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