Economical Efficiency of the AOQL Plans for Inspection by Variables and their Calculations

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Abstract—In this paper we shall deal with the AOQL single sampling plans when the remainder of rejected lots is inspected. We shall consider two types of AOQL plans - for inspection by variables and for inspection by variables and attributes (all items from the sample are inspected by variables, remainder of rejected lots is inspected by attributes). These plans we shall compare with the corresponding Dodge-Romig AOQL plans by attributes. We shall report on an algorithm allowing the calculation of these plans when the non-central t distribution is used for the operating characteristic. The calculation is considerably difficult, we shall use an original method and software Mathematica. From the results of numerical investigations it follows that under the same protection of consumer the AOQL plans for inspection by variables are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans (saving of the inspection cost is 70% in any cases).

Keywords—Acceptance sampling; AOQL plans; inspection by variables; software Mathematica

I. INTRODUCTION

Under the assumption that each inspected item is classified as either good or defective (acceptance sampling by attributes) in [1] are considered sampling plans which minimize the mean number of items inspected per lot of process average quality

$$I_s = N - (N - n) \cdot L(\overline{p}; n, c)$$
⁽¹⁾

under the condition

$$\max_{0$$

(AOQL single sampling plans), where N is the number of items in the lot (the given parameter), \overline{p} is the process average fraction defective (the given parameter), p_L is the average outgoing quality limit (the given parameter, denoted AOQL), n is the number of items in the sample (n < N), c is the acceptance number (the lot is rejected when the number of

defective items in the sample is greater than c), L(p) is the operating characteristic (the probability of accepting a submitted lot with fraction defective p - see[2]), AOQ(p) is average outgoing quality (the mean fraction defective after inspection when the fraction defective before inspection was p).

Condition (2) protects the consumer against the acceptance of a bad lot. The AOQL plans for inspection by attributes are in [1] extensively tabulated.

II. AOQL PLANS BY VARIABLES AND ATTRIBUTES

The problem to find AOQL plans *for inspection by variables* has been solved in [3] under the following assumptions:

Measurements of a single quality characteristic X are independent, identically distributed normal random variables with unknown parameters μ and σ^2 . For the quality characteristic X is given either an upper specification limit U (the item is defective if its measurement exceeds U), or a lower specification limit L (the item is defective if its measurement is smaller than L). It is further assumed that the unknown parameter σ is estimated from the sample standard deviation s.

The inspection procedure is as follows: Draw a random sample of *n* items and compute \overline{x} and *s*. Accept the lot if

$$\frac{U-\bar{x}}{s} \ge k, \text{ or } \frac{\bar{x}-L}{s} \ge k.$$
(3)

We have determine the sample size n and the critical value k. There are different solutions of this problem. In paper [3] we used for determination n and k a similar conditions as Dodge and Romig in [1].

Now we shall formulate this problem. Let us consider *AOQL plans for inspection by variables and attributes* – all items from the sample are inspected by variables, but the

remainder of rejected lots is inspected only by attributes. Let us denote

- c_s^* the cost of inspection of one item by attributes,
- c_m^* the cost of inspection of one item by variables.

Inspection cost per lot, assuming that the remainder of rejected lots is inspected by attributes (the inspection by variables and attributes), is $n \cdot c_m^*$ with probability L(p;n,k), and $n \cdot c_m^* + (N-n) \cdot c_s^*$ with probability 1 - L(p;n,k). The mean inspection cost per lot of process average quality is therefore

$$C_{ns} = n \cdot c_m^* + (N - n) \cdot c_s^* \cdot \left[1 - L(\overline{p}, n, k)\right]$$
⁽⁴⁾

Now we shall look for the acceptance plan (n,k) minimizing the mean inspection cost per lot of process average quality C_{ms} under the condition (2). The condition (2) is the same one as used for protection the consumer Dodge and Romig in [1]. Let us introduce a function

$$I_{ms} = n \cdot c_m + (N - n) \cdot \left[1 - L(\overline{p}; n, k)\right], \tag{5}$$

where

$$c_m = c_m^* / c_s^*.$$
 (6)

Since

$$C_{ms} = I_{ms} \cdot c_s^*, \tag{7}$$

both functions C_{ms} and I_{ms} have a minimum for the same acceptance plan (n,k). Therefore, we shall look for the acceptance plan (n,k) minimizing (5) instead of (4) under the condition (2).

For these AOQL plans for inspection by variables and attributes *the new parameter* c_m was defined – see (6). This parameter must be estimated in each real situation. Usually is

$$C_m > 1.$$
 (8)

Putting formally $c_m = 1$ into (5) (I_{ms} in this case is denoted I_m) we obtain

$$I_m = N - (N - n) \cdot L(\overline{p}; n, k), \tag{9}$$

i.e. the mean number of items inspected per lot of process average quality, assuming that both the sample and the remainder of rejected lots is inspected by variables. Consequently the AOQL plans for inspection by variables are a special case of the AOQL plans by variables and attributes for $c_m = 1$. From (9) is evident that for the determination AOQL plans by variables it is not necessary to estimate c_m ($c_m = 1$ is not real value of this parameter). Summary:

For the given parameters p_L , N, \overline{p} and c_m we must determine the acceptance plan (n,k) for inspection by variables and attributes, minimizing I_{ms} under the condition (2).

Solution of this problem is in the paper [3], now we shall report on an algorithm allowing the calculation of these plans. In the first place we shall solve the equation (2), in the second place we shall determine the acceptance plan (n,k) minimizing I_{ms} under the condition (2). For given sample size *n* (and given *N*, p_L) we shall look for the critical value *k* for which holds (2), i.e. (see [3])

$$M_n(k) - p_L / (1 - \frac{n}{N}) = 0.$$
⁽¹⁰⁾

Under suitable assumptions solution of the equation (10) exists and is unique – see [2]. This solution is considerably difficult (explicit formula for *k* does not exist), we must solve (10) two times numerically (in the first step we determine x_M as a solution of equation G'(x) = 0, in the second step we determine *k* as a solution (10) – see [3]).



Fig. 1. The function G'(x) for n=60 and k=2,2

From Figure 1 is evident that numerical solution of equation G'(x) = 0 depends on good first approximation X_0 . In [5] is proved that solution X_M of this equation is between

$$x_r = \frac{k}{1+A}, \qquad x_a = \frac{k + A\sqrt{k^2 - 2(1-A^2)\ln A}}{1-A^2}.$$
 (11)

Therefore we choose for X_0 following point (numerical investigations show that this point is good start value)

$$x_0 = \frac{(100+n)x_r + nx_a}{2n+100}.$$
 (12)

III. AOQL PLANS BY VARIABLES AND ATTRIBUTES – NUMERICAL SOLUTION

Analogously as for calculation of the LTPD plans (see [6]) we shall use software Mathematica for calculation of the AOQL plans for inspection by variables and attributes (see [7]).

Example. Let N=1000, p_L =0.0025, \overline{p} =0.001 and c_m =1.8 (the cost of inspection of one item by variables is higher by 80% than the cost of inspection of one item by attributes). We shall look for the AOQL plan for inspection by variables and attributes. Furthermore we shall compare this plan and the corresponding Dodge-Romig AOQL plan for inspection by attributes.

Solution. In the first step we shall determine X_M as a solution of equation G'(x) = 0. According to (11) and (12) we have (G'(x) see [3])

In[1]:= << Statistics`ContinuousDistribution`

In[2]:= ndist = NormalDistribution[0, 1]

In[3]:= cm = 1.8

In[4]:= AOQL=0.0025

In[5]:= pL = 0.0025

In[6] := pbar = 0.001

In[7]:= nbig = 1000

In[8]:= A[n_, k_] := Sqrt[1/n + k^2/(2n - 2)];

 $G'[x_n, k_] := CDF[ndist, (x - k)/A[n, k]] - CDF[ndist, -x]^*$

 $Exp[-((1 - A[n, k]^2) x^2 - 2k x + k^2)/(2A[n, k]^2)]/A[n, k];$

 $xr[n_, k_] := k/(1 + A[n, k]);$

 $xa[n_, k_] := (k + A[n, k]*Sqrt[k^2 - 2(1 - A[n, k]^2)*$

 $Log[A[n, k]])/(1 - A[n, k]^2);$

 $x0[n_k_{-}] := ((100 + n)*xr[n, k] + n*xa[n, k])/(2n + 100);$

 $FR[n_, k_] := FindRoot[G'[x, n, k] == 0, \{x, x0[n, k]\}];$

 $xM[n_, k_] := x /. FR[n, k];$

Using Newton's method for solution (10) $(M_n(k), M_n'(k))$ see [3]) with start point o=1.6 we have

```
\begin{split} c[n\_, k\_] &:= -(CDF[ndist, -xM[n, k]]*CDF[ndist, (xM[n, k] -k)/\\ & A[n, k]] - pL/(1 - n/nbig))/\\ & (-CDF[ndist, -xM[n,k]]*(1/n + k xM[n, k]/\\ & (2n - 2))*Exp[-(xM[n, k] - k)^2/(2A[n, k]^2)]/\\ & (A[n, k]^3*Sqrt[2Pi])); \end{split}
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o = 1.6;
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 $\label{eq:recAux[n_i]:=fRecAux[n,i]=fRecAux[n,i-1]+c[n,fRecAux[n,i-1]];} fRecAux[n_i,0]=o;$

k[n_]:=fRecAux[n,7];

The acceptance plan (n,k) minimizing I_{ms} (see (5)) under the condition (2) is:

 $\label{eq:a[n_]:=CDF[ndist, (k[n] - Quantile[ndist, 1 - pbar])/Sqrt[1/n + k[n]^2/(2n - 2)]];$

 $Ims[n_] := n \ cm + (nbig - n)*a[n];$

FMinSearch[nl_,nu_]:= nl /; nl == nu; FMinSearch[nl_,nu_]:= FMinSearch[nl, nl+Floor[(nu-nl)/2]] /; Ims[nl+Floor[(nu-nl)/2]]<= Ims[nl+Floor[(nu-nl)/2]+1]; FMinSearch[nl_,nu_]:= FMinSearch[nl+Floor[(nu-nl)/2]+1, nu];

n=FMinSearch[7, nbig/2];

Correction for non-central t distribution (the operating characteristic is e.g. in [4]):

```
In[25]:= lambda[p_]:=Quantile[ndist,1-p]*Sqrt[n]
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In[26]:= nonctdist[p_]:=NoncentralTDistribution[n-1,lambda[p]]

In[27]:= L1[p_]:=1-CDF[nonctdist[p], k[n]*Sqrt[n]]

 $In[28]:=AOQ[p_]:=(1-n/nbig)*p*L1[p]$

In[29]:= d=0.00001

 $In[30]:=fMSmodq[pl_,pu_]:=pl /; pl==pu$

 $fMsmodq[pl_,pu_] := fMSmodq[pl,pl+Floor[(pu-pl)/(2d)]*d] \ /;$

-AOQ[pl+Floor[(pu-pl)/(2d)]*d] <= -AOQ[pl+Floor[(pu-pl)/(2d)*d]+d]

fMSmodq[pl_,pu_]:=fMSmodq[pl+Floor[(pu-pl)/(2d)]*d+d, pu]

In[33]:= pLtrue := AOQ[fMSmodq[0.00001, 0.01]]

In[34] := pcpL = 0.00000001

 $\label{eq:In[35]:= samplan := {n, k[n]} /; (konst=pLtrue; Abs[konst - AOQL] < pcpL); \\ Samplan := (pL=pL+AOQL-konst; Clear[fRecAux]; fRecAux[n_i_i] := fRecAux[n,i] = fRecAux[n,i-1]+ c[n,fRecAux[n,i-1]]; \\ \end{cases}$

fRecAux[n_,0]=0; samplan)

In[37]:= samplan

Out[37]= {49, 2.57617}

The AOQL plan for inspection by variables and attributes is n = 49, k = 2.57617.

The corresponding AOQL plan for inspection by attributes we find in [1]. For given parameters N, p_L and \overline{p} we have $n_2 = 130$, c = 0. For the comparison these two plans from an economical point of view we use parameter e defined by relation

$$e = \frac{I_{ms}}{I_s} \cdot 100.$$

The AOQL plan for inspection by variables and attributes is more economical than the corresponding Dodge-Romig plan when

e < 100.

The Mathematica gives

In[38] := n = 49

In[39] := k = 2.57617

In[40] := n2 = 130

In[41] := c = 0

 $In[42] := L1[p_]:= 1 - CDF[nonctdist[p], k*Sqrt[n]]$

In[43] := L2[p_]:= Sum[Binomial[nbig*p, i]*

Binomial[nbig - nbig*p, n2 - i]/Binomial[nbig, n2],{i, 0, c}])

In[44] := e = 100*(n*cm+(nbig-n)*(1-L1[pbar]))/(nbig-(nbig-n2)*L2[pbar])

Out[44]:= 52.2008

Since e = 52.2008%, using the AOQL plan for inspection by variables and attributes (49, 2.57617) it can be expected approximately **48% saving of the inspection cost** in comparison with the corresponding Dodge-Romig plan (130, 0).

Further we compare the operating characteristics of these plans:

In[45]:= Table[{p, N[L1[p], 6], N[L2[p], 6]}, {p, 0.001, 0.031, 0.002}]

In[46]:= TableForm[%]

Out[46]//TableForm=

0.001	0.959306	0.87
0.003	0.734422	0.658207
0.005	0.522047	0.497674
0.007	0.365862	0.376067
0.009	0.256813	0.284003
0.011	0.181453	0.214346
0.013	0.129244	0.161675
0.015	0.0928235	0.121872
0.017	0.0672038	0.0918112
0.019	0.049026	0.0691225
0.021	0.0360195	0.0520083
0.023	0.0266387	0.039107
0.025	0.019822	0.0293876
0.027	0.0148338	0.0220699
0.029	0.0111597	0.0165638
0.031	0.00843709	0.0124235

For example we get $L_1(\overline{p}) = L_1(0.001) = 0.959306$, i.e. the producer's risk for the AOQL plan for inspection by variables and attributes is therefore approximately $\alpha = 1 - L_1(\overline{p}) = 0.04$.

The producer's risk for the corresponding Dodge-Romig plan is $\alpha = 1 - L_2 (\overline{p}) = 1 - 0.87 = 0.13$.

Finally graphic comparison of the operating characteristics of these plans (see Figure 2):

 $In[47]:= oc1 = Plot[L1[p], \{p, 0, 0.025\}, AspectRatio -> 0.9,$

AxesLabel -> {"p", "L(p)"}, PlotStyle -> $[\underline{Bhg}kness[0.0045]]$

 $In[48]:= oc2 = ListPlot[Table[\{p, L2[p]\}, \{p, 0, 0.025, 0.0003\}]]$

In[49]:= Show[oc1, oc2]



Fig. 2. OC curves for the AOQL sampling plans for inspection by variables and attributes (49, 2.57617) ______ for inspection by attributes (130, 0)

CONCLUSION

From these results it follows that the AOQL plan for inspection by variables and attributes is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan (48% saving of the inspection cost). Furthermore the OC curve for the AOQL plan by variables and attributes is better than corresponding OC curve for the AOQL plan by attributes - see Figure 2 (for example the producer's risk for the AOQL plan by variables and attributes $\alpha = 0.04$ is less than for the corresponding Dodge-Romig plan $\alpha = 0.13$).

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