Improving the Accuracy for Amplitude and Frequency of Analytical Equation of Single-ended Ring Oscillators Based on Circuit and Transistor Parameters

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Abstract-- ***The analytical relationships presented for amplitude and frequency of the ring oscillator are derived approximately due to the nonlinear nature of this oscillator. In the case where the transistors experience the cut-off region, the relationships presented so far have no connection between the frequency and the dimensions of the transistor, which is not valid in practice. In this paper, considering the circuit's governing equation and the ring oscillator's output waveform, a relation for the frequency is presented, including the dimensions of the transistor. Also, a simple and approximately accurate relationship for the oscillator amplitude is provided in this case. The validity of these relationships has been investigated by analyzing and simulating a single-ended oscillator in 0.18µm technology.***

Keywords- **Amplitude analytical equation, Frequency analytical equation, Nonlinear delayed differential equation, Ring oscillator, Voltage Controlled Oscillator**

I. INTRODUCTION

Ring oscillators are continually one of the main options for designing oscillator circuits due to their low power consumption, relatively wide tuning range, and low die area due to the lack of inductors. The design of a ring oscillator mainly includes a trade-off between speed, power consumption, tuning range, phase noise, etc. It is necessary to know information about design variables to make this trade-off at the design level, particularly the factors that influence the frequency and amplitude of oscillators. But despite all the advantages mentioned, the nonlinear nature of this oscillator has made a significant challenge to the realization of a closed-form relationship for its amplitude and frequency. In recent decades, numerous approximate relationships for oscillator amplitude and frequency have been proposed. With assumptions such as the small-signal linear model for an inherently nonlinear ring oscillator, some have achieved analytical relationships for amplitude and frequency[1]–[5]. These relationships are not very accurate due to the linear assumption or can only be used in a particular technology, or are independent of the parameters of the technology and the dimensions of the transistor.

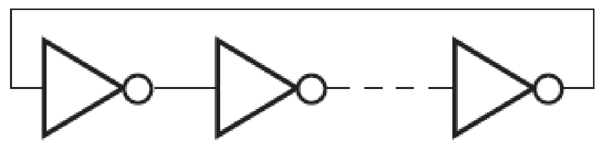

Fig. 1: Ring oscillator with N inverting stages

In this paper, Section II provides an overview of some works on the ring oscillator's amplitude and frequency calculations. Section III describes the calculation method of this paper, and finally, Section IV will compare the obtained relationships with previous methods and simulations.

II. LITERATURE REVIEW OF FREQUENCY AND AMPLITUDE EQUATIONS

A ring oscillator consists of odd numbers of generally identical inverting blocks such that the output of the last block is the input of the first block (Fig. 1). The circuit must satisfy Barkhausen's criteria to initiate oscillation; total phase shift and the feedback loop gain must be 2π and one, respectively. If the oscillator consists of N blocks, each block must have a phase shift of π/N . The remaining π phase shift is provided by DC feedback. Since the delay between the input and output of each block is t_d , the oscillation frequency can be obtained based on the delay. According to what has been stated, we have:

$$f = \frac{1}{2N.t_d} \tag{1}$$

The oscillation frequency accuracy depends on the accuracy of the t_d . The main problem in calculating t_d is the nonlinear nature of the ring oscillator blocks. That is why several studies have been done on this subject[3], [4]. Some of these methods used to calculate t_d are reviewed in [6]. Despite the low accuracy of the simplifications used to achieve the closed-form relationship to calculate the delay propagation time, the details of these relationships make it difficult to easily understand the role of circuit and transistor parameters on the frequency. In [7], relationships for amplitude and frequency are obtained using the equations governing the oscillator and based on the output

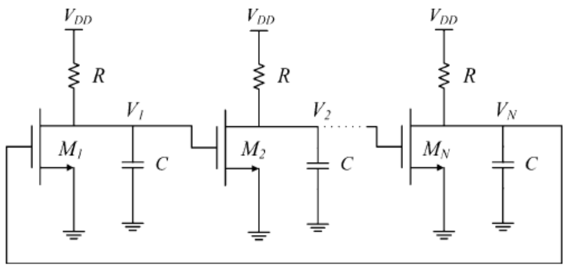

Fig. 2: Single-ended, N-stages ring oscillator circuit

waveform of the oscillator. Still, modeling the transistor with the ideal switch ignores the role of the transistor's parameters in amplitude and frequency. The obtained frequency and amplitude relationship are implicitly proportional to each other and only proportional to the resistor, capacitor, and the number of inverting blocks. In [8], assuming the approximation of the output waveform of each block with the $\tanh(.)$ function, closed-form amplitude and frequency relationships were extracted, which is valid for three- and five-stage oscillators.

According to the cases studied, this paper presents a method to obtain a more accurate relationship for the oscillation frequency proportional to the specifications of the circuit and the transistor.

III. PROPOSED METHOD

A simple ring oscillator is shown in Fig. 2. If the output of the oscillator is voltage V_1, we have:

$$\frac{V_1 - V_{DD}}{R} + C\frac{dV_1}{dt} + i_{d_1} = 0 \tag{2}$$

where the i_{d_1} is as follows:

$$i_{d_1} = f\left(V_N\left(t\right)\right) = f\left(V_1\left(t-\tau\right)\right) \tag{3}$$

and the function $f(.)$ depends on the operating region of the transistor and τ is delay between consecutive inverting blocks.

Equation (2) is a nonlinear delayed differential equation that generally does not have an explicit and closed-form answer and is often analyzed by numerical methods.[9] Therefore, obtaining a closed-form relationship for the ring oscillator parameters such as amplitude and frequency based on the circuit parameters has not been possible accurately and has always been a problem and challenge in the design of ring oscillators. The output waveform of a ring oscillator will vary depending on what operating region the transistors experience while oscillating. Fig. 3 shows four of these waveforms. The oscillator's design in the saturation region or to experience only the saturation and triode regions, both because the range of transistor dimensions and resistor is limited and because it has a low oscillation amplitude generally not welcomed by designers. This is why most ring oscillator outputs are in the form of Fig. 3-d. Therefore, this paper provides a method for calculating the amplitude and frequency of the oscillator when transistors experience all operating regions.

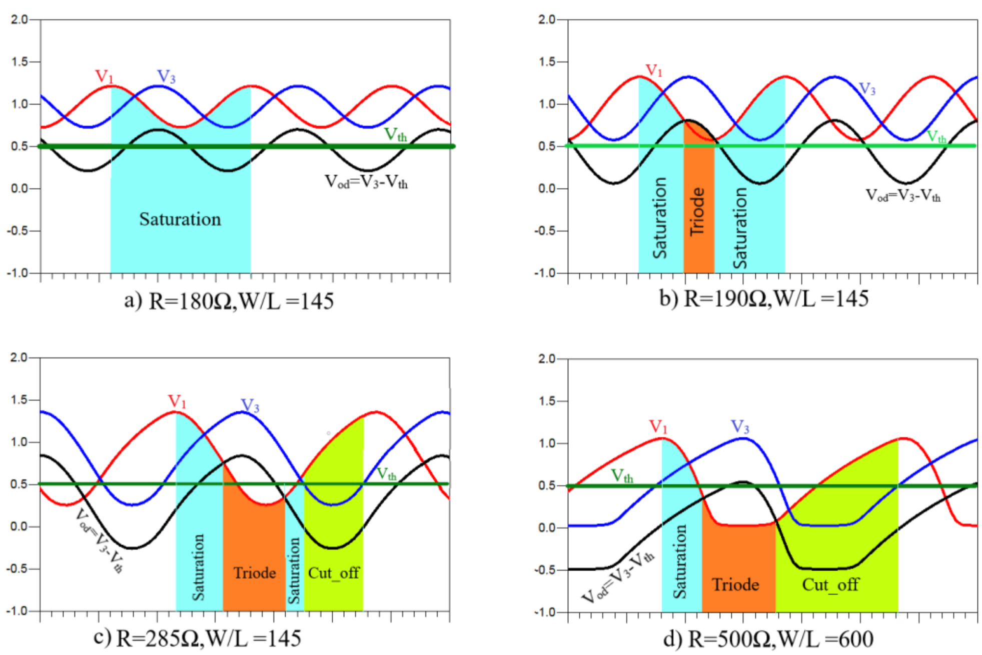

Fig. 3: Ring oscillator output waveform for $V_{DD} = 1.8v, L = 0.18um$ *and different resistor and width*

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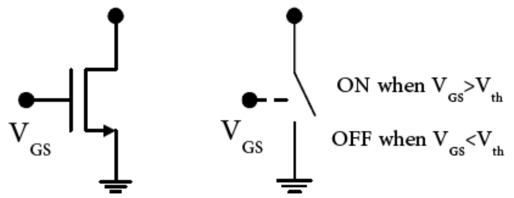

Fig. 4: Transistor model as an ideal switch

To better understand the performance of an oscillator for deriving relationships, we assume that the oscillator transistors in Fig. 2 are ideal switches (Fig. 4). In this case, the output waveform of a 5-stage oscillator will be in Fig. 5. Therefore, when $V_{GS} < V_{th}$, the transistor is off and is equivalent to an open switch. In this case, the capacitor C is charged by V_{DD} through the resistor. After V_{GS} reaches V_{th}, the drain-source of the transistor is short-circuited, and the capacitor is discharged. So, the output of the first stage is zero, and the second stage capacitor starts charging. When the second stage voltage reaches V_{th}, the third stage capacitor is discharged, and the fourth stage capacitor begins to charge. The waveform shown in Fig. 5 will be obtained by following this process.

If the oscillator consists of $N = 2k+1$ stages with an ideal switch and the output period of the oscillator is T, the transistor of each stage is on for kT/N (output voltage is zero) and for $(k+1)T/N$ is off (capacitor is on charge). The voltage relation of the capacitor of each stage is as follows:

$$v_c(t) = V_{DD}\left(1 - e^{-\frac{t}{RC}}\right) \tag{4}$$

If we consider the voltage of the second stage as a basis, after T/N has passed from the instant of starting the charge of the capacitor of the second stage, the voltage of this stage reaches the value of V_{th} (Fig. 5). Consequently:

$$V_{th} \cong V_{DD}\left(1 - e^{-\frac{T/N}{RC}}\right) \Rightarrow T_{Ideal} = NRC\ln\left(\frac{V_{DD}}{V_{DD} - V_{th}}\right) \tag{5}$$

Where T_{Ideal} is the oscillator output period if the transistor is modeled by an ideal switch. Since the capacitor of each block is charging during $t = (k+1)T/N$ time, the maximum output voltage of the oscillator is ideally equal to:

$$V_{\max} = V_{DD}\left(1 - e^{-\frac{(k+1)T/N}{RC}}\right) \tag{6}$$

By combining the recent relation with (5), we have:

$$V_{\max} = V_{DD}\left(1 - \left(\frac{V_{DD} - V_{th}}{V_{DD}}\right)^{(k+1)}\right) \tag{7}$$

Equation (7) provides the oscillation amplitude based on the number of oscillator stages, V_{DD} and V_{th}. As k increases, the

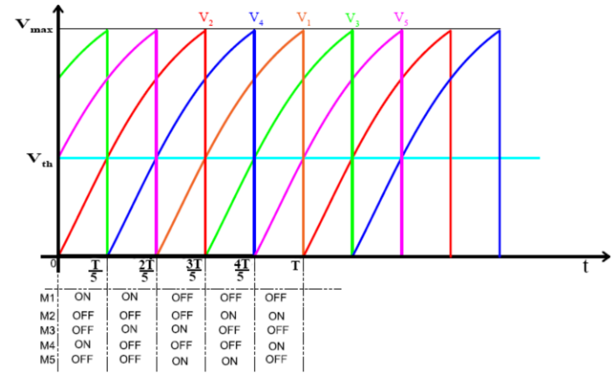

Fig. 5: 5-stage ring oscillator output with ideal transistors

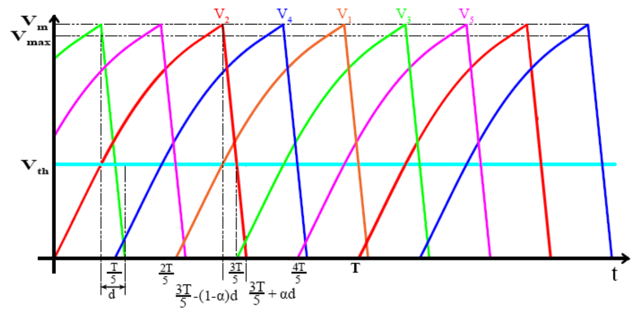

Fig. 6: 5-stage ring oscillator output with real transistor model

output period increases, resulting in a longer capacitor charge time, which brings the $V_{\max}$ value closer to the V_{DD}.

However, in practice, the transistor is not as ideal as the switch when it is on, and it has a resistance, so the capacitor discharge operation when the transistor is turned on requires a time interval of d. This issue will decrease the frequency of the oscillator output waveform. In this case, the capacitors' waveform is the same as in Fig. 6. In this case, the transistor has a resistance when turned on (Fig. 7). When the transistor turns on, the current of the transistor is less than the current drawn from the voltage source V_{DD}. Consequently, the capacitor is still charged, and then the capacitor is discharged (Fig. 6 And Fig. 8). Therefore, the maximum voltage value is greater than the value calculated in (6). Still, this value is considered equal to the maximum voltage value in the ideal state for simplicity (Eq (7)).

On the other hand, due to the low value of the resistor of the transistor when it is on compared to the R, the discharge curve of the capacitor can be approximated as a straight line. This situation is shown in Fig. 8 for the second capacitor voltage in a 5-stage oscillator. In general, the capacitor is discharged in the $\frac{k+1}{N}T - (1-\alpha)d \le t \le \frac{k+1}{N}T + \alpha d$ interval for an N-stage oscillator. The value of α according to the linear approximation for the discharge step is equal to:

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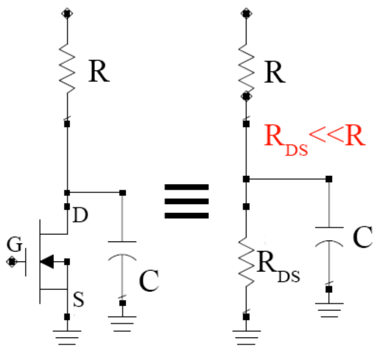

Fig. 7: Real transistor model, when turned on

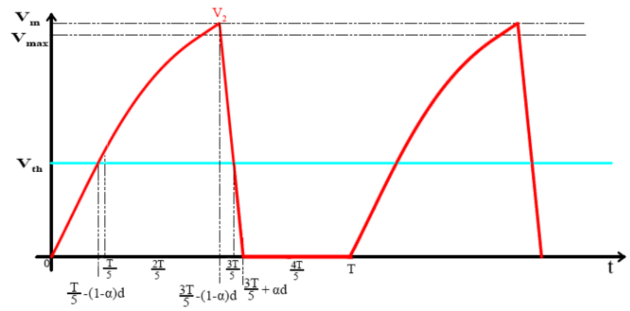

Fig. 8: Second stage output of the oscillator in real

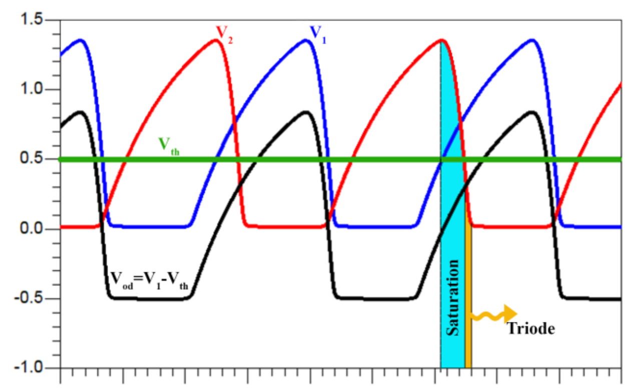

Fig. 9: Oscillator output status when capacitor discharge

$$\alpha = \frac{V_{th}}{V_{\max}} \tag{8}$$

Furthermore, the passing point of V_{th} in the capacitor charging phase is equal to:

$$t = \frac{T}{N} - (1-\alpha)d \tag{9}$$

Therefore:

$$\begin{gathered} V_{th} \cong V_{DD}\left(1 - e^{-\frac{T/N-(1-\alpha)d}{RC}}\right) \Rightarrow \\ \frac{T}{N} - (1-\alpha)d = RC\ln\left(\frac{V_{DD}}{V_{DD}-V_{th}}\right) \end{gathered} \tag{10}$$

When the capacitor is discharged during d, the transistor is on and is in the saturation region for most of the discharge interval (Fig. 9). In the relations presented to calculate the oscillation frequency of the ring oscillator[7],[10]–[12], a vertical line approximates this part of the output waveform. This approximation reduces the accuracy of the calculations, leading to relationships for the oscillator frequency independent of the transistor's dimensions and technology's parameters. To obtain the equivalent resistance of the transistor in this interval according to Fig. 7 and using the current relationship of long channel transistors, we have:

$$I_D = \frac{V_2}{R_{DS}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS_2} - V_{th}\right)^2 \tag{11}$$

Where R_{DS} is the transistor's resistance at the capacitor's discharge duration (Fig. 7). Since the value of the R_{DS} is a very small value relative to the R, the discharge curve of the capacitor can be approximated by a line compared to the charge step. Also, the value of the minimum output voltage of each block is slightly higher than zero, considered zero. The capacitor voltage reaches the value of V_{th} with a delay, so the transistor of the next block is turned off with a delay. Consequently, the next capacitor starts charging with a delay than the ideal state. By approximating the capacitor charge duration curve with a line passing through the points $(0, V_{th})$, the relation of V_{GS} can be reached.

The relationship between the voltage of different stages based on the voltage of the second stage is as follows:

$$\begin{gathered} V_{2r}(t) = V_2\left(t - (r-1)\frac{T}{N}\right) \\ V_1(t) = V_{2k}\left(t - \frac{T}{N}\right) = V_2\left(t - k\frac{T}{N}\right) \\ V_{2r+1}(t) = V_1\left(t - r\frac{T}{N}\right) = V_2\left(t - (k+r)\frac{T}{N}\right) \end{gathered} \tag{12}$$

Where $1 \le r \le k$. According to (12), we have:

$$V_1(t) = \frac{V_{th}}{\frac{T}{N} - (1-\alpha)d}\left(t - \frac{kT}{N}\right) \qquad \frac{k}{N}T \le t \le T - (1-\alpha)d \tag{13}$$

On the other hand, we have:

$$V_{GS_2} = V_1(t) = V_2\left(t - \frac{kT}{N}\right) \tag{14}$$

To get the value of d, we use the value of V_{GS} at $t = \frac{(k+1)T}{N}$:

$$\begin{gathered} V_{GS_2}\left(t = \frac{(k+1)T}{N}\right) = \frac{V_{th}}{\frac{T}{N} - (1-\alpha)d}\left(\frac{(k+1)T}{N} - \frac{kT}{N}\right) \\ \frac{k}{N}T \le t \le T - (1-\alpha)d \end{gathered} \tag{15}$$

$$V_{GS_2} - V_{th} = \frac{(1-\alpha)d}{\frac{T}{N} - (1-\alpha)d} V_{th} \tag{16}$$

We also have at $t = \frac{(k+1)T}{N}$:

$$V_2\left(t = \frac{(k+1)T}{N}\right) = V_{th} \tag{17}$$

Putting the value obtained in (15) and (16) in (11), we have:

$$R_{DS} = \frac{\left[\frac{T}{N} - (1-\alpha)d\right]^2}{\frac{1}{2}\mu_n C_{ox} \frac{W}{L} V_{th} (1-\alpha)^2} \frac{1}{d^2} \tag{18}$$

By combining (10) and (18), we have:

$$R_{DS} = \frac{\left[RC \ln\left(\frac{V_{DD}}{V_{DD} - V_{th}}\right)\right]^2}{\frac{1}{2}\mu_n C_{ox} \frac{W}{L} V_{th} (1-\alpha)^2} \frac{1}{d^2} \tag{19}$$

Considering the complete discharge of the capacitor in the interval $\frac{(k+1)}{N}T - (1-\alpha)d \le t \le \frac{(k+1)}{N}T + \alpha d$ and because in the first-order RC circuit, the capacitor is almost completely discharged in 5-time constants:

$$5R_{ds}C \cong d \tag{20}$$

Therefore, using (19), we have:

$$d = \sqrt[3]{5C \frac{\left[RC \ln\left(\frac{V_{DD}}{V_{DD} - V_{th}}\right)\right]^2}{\frac{1}{2}\mu_n C_{ox} \frac{W}{L} V_{th} (1-\alpha)^2}} \tag{21}$$

Finally, by placing (21) in (10), we have:

$$T \cong N \left[RC \ln\left(\frac{V_{DD}}{V_{DD} - V_{th}}\right) + (1-\alpha) \sqrt[3]{5C \frac{\left[RC \ln\left(\frac{V_{DD}}{V_{DD} - V_{th}}\right)\right]^2}{\frac{1}{2}\mu_n C_{ox} \frac{W}{L} V_{th} (1-\alpha)^2}} \right] \tag{22}$$

The recent relation expresses the oscillator period as a function of circuit specifications (V_{DD}, R, C, N), fabrication technology (μ_n, C_{ox}, V_{th}), and transistors characteristics (W, L).

IV. RELATIONSHIPS EVALUATION

Simulation in ADS software and $0.18\mu m$ technology has been used to evaluate (7) and (22). Given that (7) is independent of the values R, C and W/L; first, the effect of these variables on the amplitude of the oscillation is investigated. Fig. 10 shows that the changes of these variables on the oscillation amplitude are very small and negligible, indicating that these variables' independence of (7) is consistent with the simulation. Applied approximations and ignoring the secondary effects cause the simulation values and (7) to differ. The magnitude of this discrepancy is shown in Table 1, which, despite the simplicity of the relationship and the approximations applied, has an acceptable error.

To evaluate (22), the oscillation frequency changes of 3, 5, and 7 stages for the R, C and W/L changes are investigated. Frequency changes are evaluated in terms of $C \in [10p, 100p, 1n, 2n]F, R = 700\Omega, W/L = 800$ in Fig. 11. In Fig. 12, this evaluation is performed in terms of resistor changes for $C = 100pF, R \in [500, 600, 700, 800]\Omega, \frac{W}{L} = 800$ and Fig. 13 in terms of transistor dimensions changes for $C = 100pF, R = 700\Omega, \frac{W}{L} \in [600, 700, 800, 1000]$. As clearly seen from these figures, the relationship presented in this paper is in excellent agreement with the simulation results.

Also, in Fig. 14 and Fig. 15, (7) and (22) are compared with the relations presented for amplitude and frequency in [6] for N=3, respectively. This comparison shows that the relationships presented in this article, in addition to being explicit, also have better accuracy.

Table 1: Comparison of the amplitude obtained from (7) and simulation

Number of stages	Amplitude [V]		Error%
N=3	Eq. (7)	0.883	11.3
	simulation	0.995	
N=5	Eq. (7)	1.15	14.2
	simulation	1.34	
N=7	Eq. (7)	1.33	13.6
	simulation	1.54	

V. CONCLUSION

Ring oscillators' Transistors generally experience all working areas of the transistor (saturation, triode, cut-off). In this paper, explicit relations for the frequency and amplitude of the oscillator are extracted using the output waveform of the oscillator. The presented frequency relationship, unlike previous works, is proportional to the dimensions and characteristics of the transistor and fabrication technology. The relationships obtained for the 3, 5, and 7-stage oscillators in 0.18µm technology are compared with the simulation results and one of the previous works in this case. It has been shown that these relationships also have good accuracy in addition to being explicit. The extracted relationships can help the designer with hand calculations. Still, as future work, they can also be used to calculate and analyze other features of the ring oscillator, such as phase noise and jitter.

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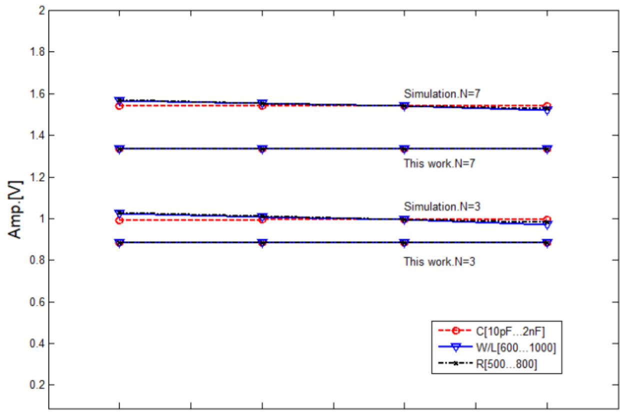

Fig. 10: Comparison of amplitude relationship with simulation for resistor, capacitor, and W/L changes

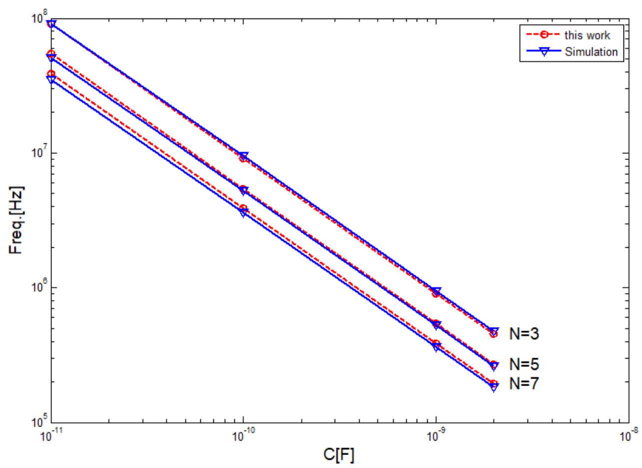

Fig. 11: Comparison of amplitude relationship with simulation for capacitor changes

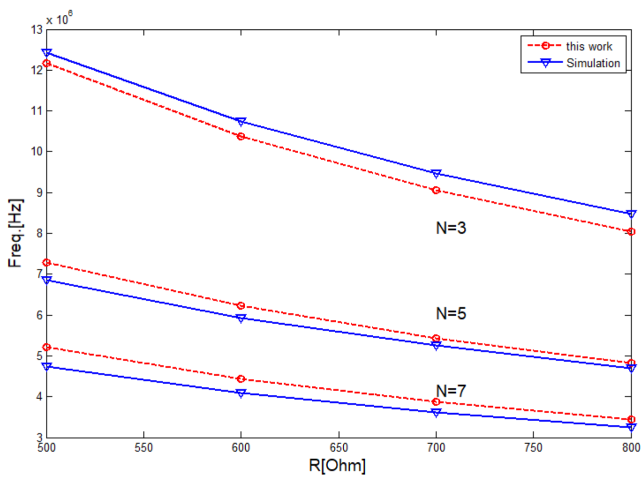

Fig. 12: Comparison of amplitude relationship with simulation for resistor changes

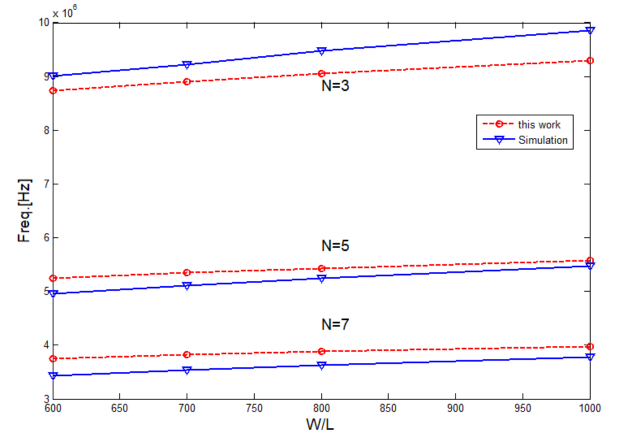

Fig. 13: Comparison of frequency relationship with simulation for changes in transistor dimensions

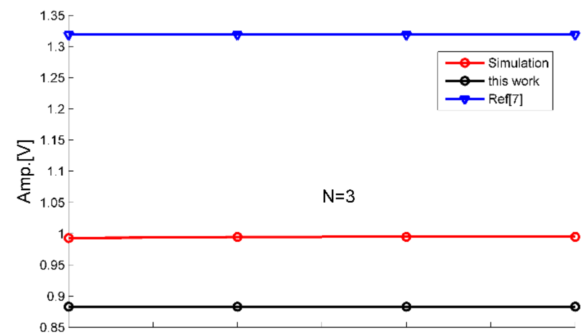

Fig. 14: Comparison of simulation and analytical result of this work and Ref[7] for oscillation amplitude

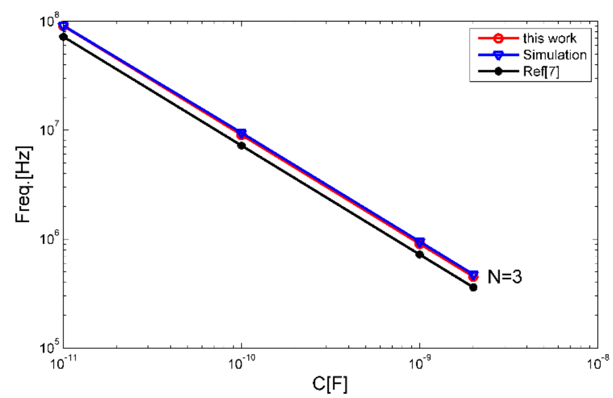

Fig. 15: Comparison of simulation and analytical result of this work and Ref[7] for oscillation frequency

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Contribution of authors:

Mehrdad Moradnezhad carried out the relations and simulations and wrote the paper.
Hossein Miar-Naimi contributed analysis tools and revised the results.

Data Availability Statement:

"No new data were created during the study."

Declaration of interests:

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.