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A Practical Method for Estimating Mutual Inductance in Wireless Power Transmission System

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Abstract- This paper proposes a practical method for estimating mutual inductance in wireless power transmission system based on the principle of electromagnetic induction. Conventional estimation methods utilize a voltage, current, and phase of current in transmitter side, while our method requires only an amplitude of the voltage and current in transmitter side. Our method is designed so that the mutual inductance can be estimated under challenging situation which there is a measurement noise. Numerical simulations show that if the system has a small mutual inductance, a relative error and standard derivation of estimated mutual inductance tend to become larger. However, it will be shown that these factors can be improved to make a voltage of voltage source in the system high.

Keywords- Free coil position, Mutual inductance estimation, Least square method, Wireless power transmission

I. Introduction

A wireless power transmission (WPT) system based on the principle of electromagnetic induction is utilized for charging mobile devices such as smart phones[1]. Recently, the system is expected to apply widely to devices such as implant devices[2], electric vehicles[3]-[5] and underwater vehicles[6]-[8] for supplying an electric power wirelessly. In application of wireless power transmission technology to the electric vehicles, there are studies for improving usability such as fast charging[4] and charging while the electric vehicles are running which is introduced in [3] and for human safety due to emitted magnetic field by WPT system[5]. For charging wirelessly an implant device, the embed WPT system will be relatively smaller than the other system. Under this situation, [2] presents a system configuration and its calculation for improving the transmission efficiency. In the WPT system which works under existence of seawater rather than air, the transmission efficiency takes account of the eddy current loss, which is analyzed in [6] and [7]. [8] optimizes coils design to reduce some losses such as eddy current loss, resistive loss of copper for the coils, etc. and maximize mutual inductance between the coils.

The system has coils at least each transmitter and receiver side for supplying the power and may decrease transmitting efficiency and power if the relative position of the coils is changed[9]. Therefore, to avoid decreasing them, there are some mechanical approaches to suppressing the change of relative position, which is introduced in the literature [10]: the first approach is that the transmitting and receiving coils are guided to be coaxial by

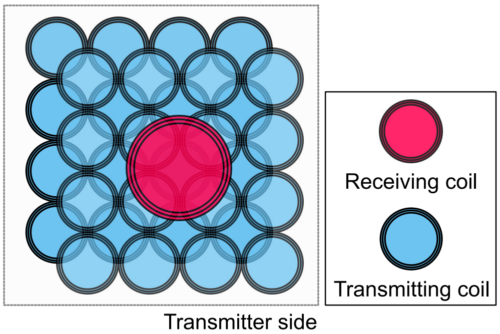

Fig. 1: An image of the third approach

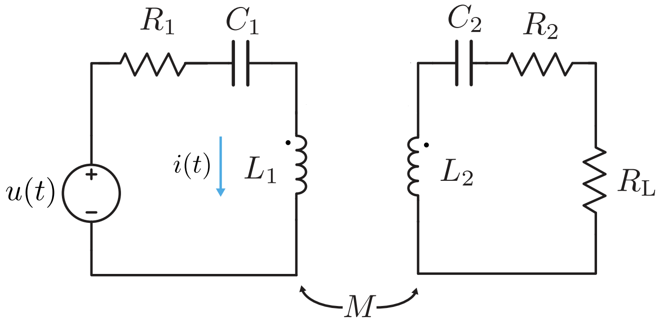

Fig. 2: A circuit model of WPT system

magnets. The second approach is that the transmitting coil moves near the receiving coil. The third approach is that transmitting coils close to the receiving coil are utilized in the transmitting coils which are fully laid on the transmitter side, which is shown in Fig. 1. These approaches keep virtually the relative position, and therefore it is fixed, and especially the first and second approaches are hard to be applied for supplying power to running EVs for instance. Furthermore, for example, in a situation of wireless charging to underwater vehicles on seawater, transmitter side may be arranged on a harbor, a ship, or a buoy. The vehicles which are receiver side cannot remain at a same position due to sea waves and currents. This means that the relative position between transmitting and receiving coil may be changed each time.

While there are approaches to improving the transmitting power and efficiency under free relative position of the coils. They are based on capturing a change of mutual inductance (or coupling coefficient) as a change of the relative position and estimate the mutual inductance (or coupling coefficient) by a voltage and current in transmitter [11][12] and receiver[13][14] side.

This paper presents a method for estimating a mutual inductance by a voltage and current in transmitter side. Our method requires only an amplitude of voltage and current in transmitter side, while [11][12] require the amplitude of voltage, current, and phase of current. Additionally, our method can perform the estimation even if a measured current contains a certain measurement noise which is approximately 31.8 dB in the average of a signal to noise ratio according to the result of the verification in the section V. Especially, this kind of the noise may be seen remarkably in a practical WPT system: for example, the current is measured by a relatively small voltage of resistor that has small resistance to avoid wasting power consumption inserted the transmitter side.

Therefore, under the noise, some numerical simulations verify the relative error of estimated mutual inductance by our method. Furthermore, this paper also presents how the system should be improved for reducing relative error.

We note that in the literature [13], their estimation method also considers a noise and is based on the recursive least square method. Our estimation method is based on the least square method, and however, there are two differences : a voltage and current that are required for the estimation, we mention above, and system improvement is proposed for an estimation performance.

II. Preparation

We assume that the WPT system can be described in a circuit shown in Fig. 2 where the voltage source outputs sinusoidal voltage with time t [s] and an angular frequency ω [rad/s], and R_1, R_2, and R_L are an internal resistance [Ω] in transmitter and receiver side and a resistive load [Ω] respectively. We also assume circuit parameters are known except a mutual inductance M [H]. We note that

$$L_1L_2 - M^2 > 0,\ M > 0. \tag{1}$$

Let $G(j\omega)$ be a frequency transfer function from a voltage of voltage source $\hat{u}(j\omega)$ to current in transmitter side $\hat{i}(j\omega)$, where $\hat{\ }$ expresses a laplace transformed amount, and j is an imaginary unit. An amplitude of the current can be written in

$$|\hat{i}(j\omega)| = |G(j\omega)||\hat{u}(j\omega)|, \tag{2}$$

where

$$|G(j\omega)| = \frac{x_6\sqrt{x_1^2+\omega^2x_2^2}}{\sqrt{x_3^2+(x_1-x_6\omega(x_4M^2+x_5))^2}}, \tag{3}$$
$$x_1 = 1 - C_2L_2\omega,$$
$$x_2 = C_2(R_2+R_L),$$
$$x_3 = \omega^2(C_1R_1x_1 + x_2 - C_1L_1\omega^2x_2),$$
$$x_4 = C_2\omega^2,$$
$$x_5 = L_1x_1 + R_1x_2,$$
$$x_6 = C_1\omega.$$

III. Proposed method for estimating mutual inductance

Assuming that a voltage amplitude of voltage source $|\hat{u}(j\omega)|$ is given large enough, and the current amplitude $|\hat{i}(j\omega)|$ is measurable and contains measurement noise. We measure $|\hat{i}(j\omega)|$ several times and perform parameter fitting to (2) for the mutual inductance M by the measured $|\hat{i}(j\omega)|$s. Then an estimated mutual inductance $\tilde{M}$ can be obtained by parameter fitted (2) for the M. A criterion of the parameter fitting is choosing $M = \tilde{M}$ to minimize a sum of square of residual between the measured $|\hat{i}(j\omega)|$ and (2). That is, under (1), we calculate

$$\tilde{M} = \underset{M}{\operatorname{argmin}}\, Q,$$
$$Q = \sum_{p=1}^{N} \left(|\hat{i}(j\omega_p)|_p - |G(j\omega_p)||\hat{u}(j\omega_p)| \right)^2 \quad (4)$$

where N is a total number of measurements, and a subscript p describes each measurement.

We note that the methods proposed in [11] and [12] for estimating the mutual inductance from transmitter side under circuit parameters which are known is performed by a voltage of voltage source, a current through the transmitter, and a phase between the current and the voltage, while our method does not require the phase.

IV. Verification procedure

In order to verify our method, we evaluate a relative error of estimated mutual inductance $\tilde{M}$ with a signal to noise ratio (SNR) that is defined latter by numerical simulations as an estimation performance of $\tilde{M}$. An evaluation procedure is the following:

1) In Fig. 2 with circuit parameters shown on Table 1, we give ω, $|\hat{u}(j\omega)|$, and M and calculate $i(t)$ until N_pth period. We note that the current $i(t)$ is in a steady state. We make a numerical sequence $s[k]$ ($k = 1, 2, \cdots, N_sN_p$) that is sampled with N_s points per a period from the $i(t)$.

2) We make a pseudo random sequence $n[k]$ that has an approximate normal distribution with average 0 [A] and standard deviation σ [A] as a measurement noise.

3) We obtain $y[k] = s[k] + n[k]$ as a measured $i(t)$ in a practical WPT system, and we calculate that a current amplitude $|\hat{i}(j\omega)| = \frac{\max(y[k]) + |\min(y[k])|}{2}$.

4) We repeat the above procedure N times and estimate a mutual inductance by (4) and evaluate a relative error of the estimated mutual inductance $\tilde{M}$.

Pseudo random sequence $n[k]$ is generated by Mersenne twister[15] , and (4) is performed by Nelder-mead method[16]. The other setup is $N_s = 20$, and $N_p = 1$. A frequency of voltage source f is adjusted to maximize almost a power of load and is varied with $f =$ 106.0, 106.5, 107.0, 107.5, and 108.0 [kHz]. We mention σ and $|\hat{u}(j\omega)|$ later.

Table 1: Circuit constants in the verification

Parameter	Value
R_1	3.00 [Ω]
R_2	1.87 [Ω]
R_L	1.02 [Ω]
C_1	21.1 [nF]
C_2	80.0 [nF]
L_1	104 [μH]
L_2	107 [μH]

We define the signal to noise ratio SNR:

$$\mathrm{SNR} = 20\log_{10}\left(\frac{\sqrt{\frac{1}{N_sN_p}\sum_{k=1}^{N_sN_p} s[k]^2}}{\sqrt{\frac{1}{N_sN_p}\sum_{k=1}^{N_sN_p} n[k]^2}}\right) \text{ [dB]}.$$

The verification has two scenarios:

1) Scenario 1

We give a mutual inductance M [H] and evaluate a relative error of $\tilde{M}$ with the procedure under fixed $\sigma = 0.08$ [A] and $|\hat{u}(j\omega)| = 20.0$ [V].

2) Scenario 2

We verify whether a change of the $|\hat{u}(j\omega)|$ and σ can reduce the relative error of $\tilde{M}$.

We verify an estimation performance of our method by these two scenarios. We note that each numerical simulation is repeated 100 times.

V. Verification Result of Scenario 1

We estimate the mutual inductance for cases of $M =$ 5.00, 10.0, and 15.0 [μH] with fixed $\sigma = 0.08$ [A] and $|\hat{u}(j\omega)| = 20.0$ [V].

A. The case of $M = 5.00$ [μH]

In the case of $M = 5.00$ [μH], results of $\tilde{M}$, relative errors of $\tilde{M}$, and averages of SNR are shown in Fig. 3, Fig. 4, and Fig. 5 respectively for each numerical simulation. There are statistics for each result on Table 2, Table 3, and Table 4 respectively.

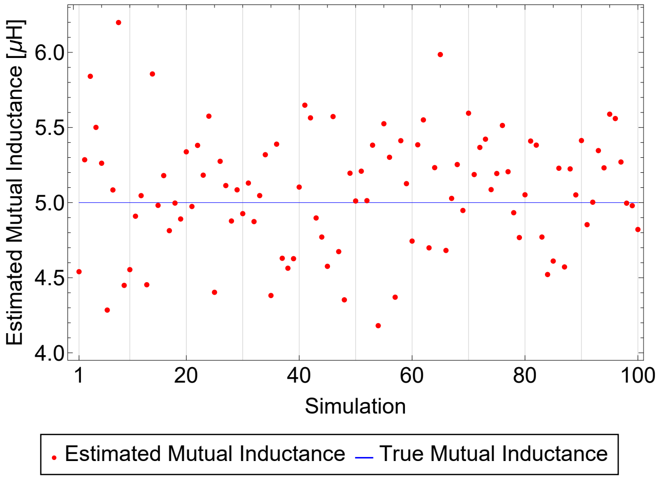

Fig. 3: The result of estimated mutual inductance $\tilde{M}$s for each simulation

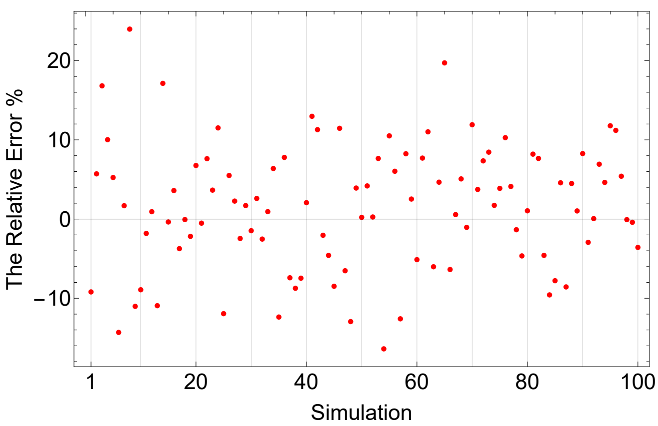

Fig. 4: The relative errors of estimated mutual inductance $\tilde{M}$s for each simulation

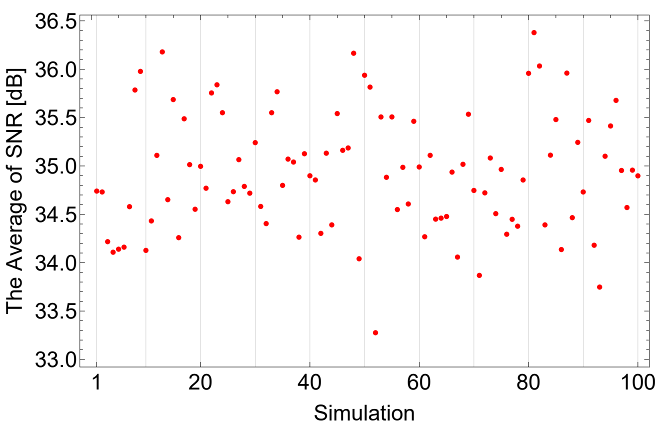

Fig. 5: The average of SNRs for each simulation

Table 2: A statistic for estimated mutual inductance $\tilde{M}$s

Average [μH]	Standard deviation [μH]	Minimum [μH]	Maximum [μH]
5.08	0.393	4.18	6.20

Table 3: A statistic for the relative errors

Average %	Standard deviation %	Minimum %	Maximum %
1.55	7.87	-16.4	24.0

Table 4: A statistic for the avergae of SNRs

Average [dB]	Standard deviation [dB]	Minimum [dB]	Maximum [dB]
34.9	0.610	33.3	36.4

B. *The case of* $M = 10.0$ [μH]

In the case of $M = 10.0$ [μH], results of $\tilde{M}$, relative errors of $\tilde{M}$, and averages of SNR are shown in Fig. 6, Fig. 7, and Fig. 8 respectively for each numerical simulation. There are statistics for each result on Table 5, Table 6, and Table 7 respectively.

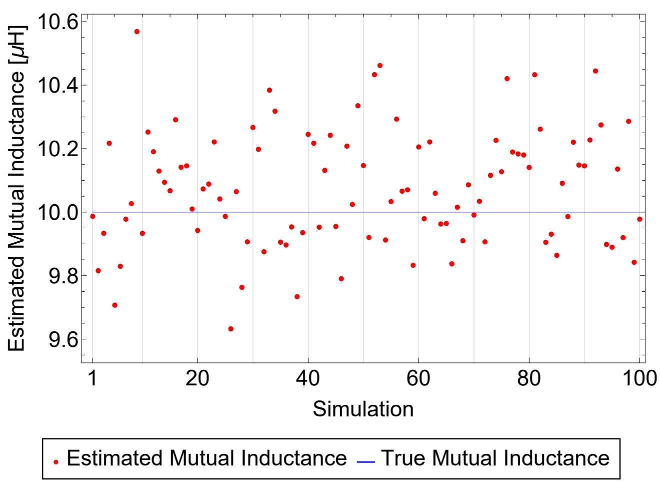

Fig. 6: The result of estimated mutual inductance $\tilde{M}$s for each simulation

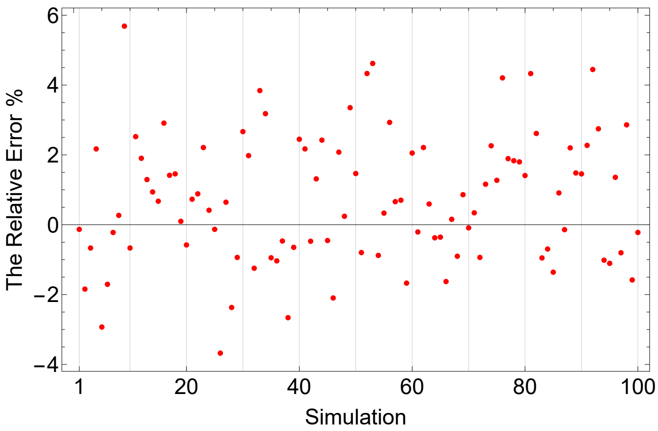

Fig. 7: The relative errors of estimated mutual inductance $\tilde{M}$s for each simulation

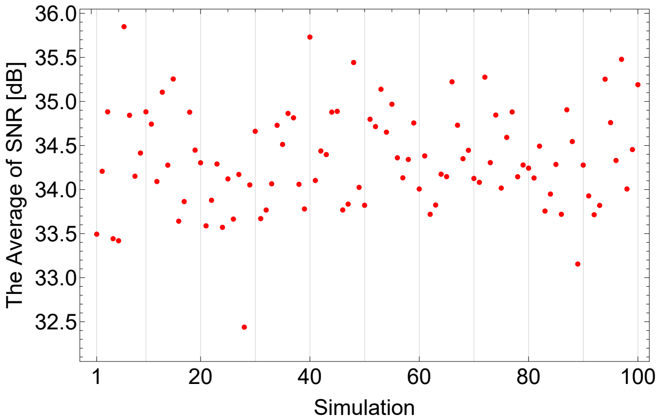

Fig. 8: The average of SNRs for each simulation

Table 5: A statistic for estimated mutual inductance $\tilde{M}$s

Average [μH]	Standard deviation [μH]	Minimum [μH]	Maximum [μH]
10.1	0.183	9.63	10.6

Table 6: A statistic for the relative errors

Average %	Standard deviation %	Minimum %	Maximum %
0.741	1.83	-3.68	5.69

Table 7: A statistic for the avergae of SNRs

Average [dB]	Standard deviation [dB]	Minimum [dB]	Maximum [dB]
34.3	0.573	32.4	35.8

C. The case of $M = 15.0$ [μH]

In the case of $M = 15.0$ [μH], the results of $\tilde{M}$, relative errors of $\tilde{M}$, and averages of SNR are shown in Fig. 9, Fig. 10, and Fig. 11 respectively for each numerical simulation. There are statistics for each result on Table 8, Table 9, and 10 respectively.

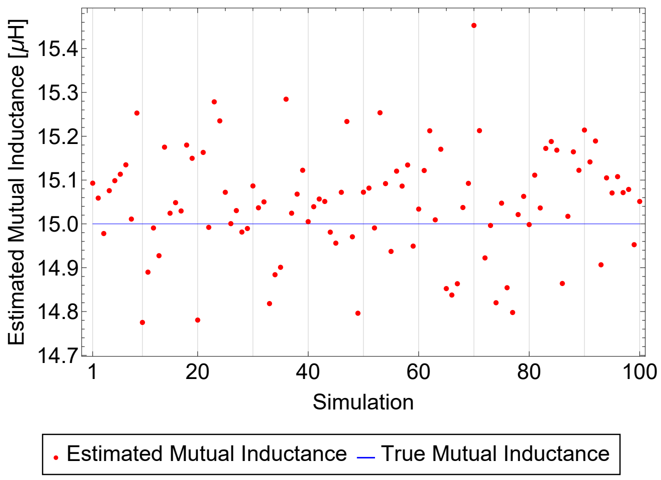

Fig. 9: The result of estimated mutual inductance $\tilde{M}$s for each simulation

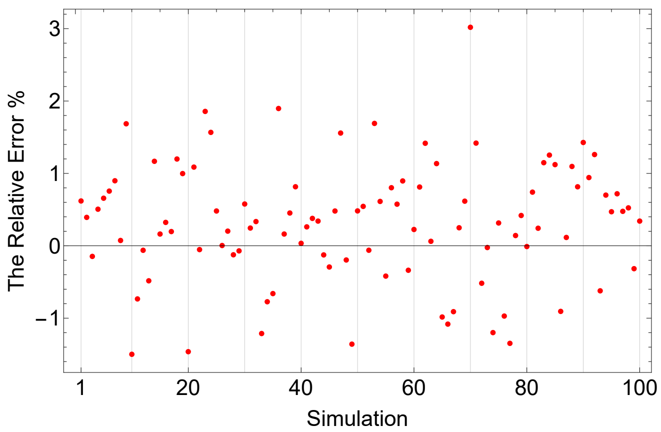

Fig. 10: The relative errors of estimated mutual inductance $\tilde{M}$s for each simulation

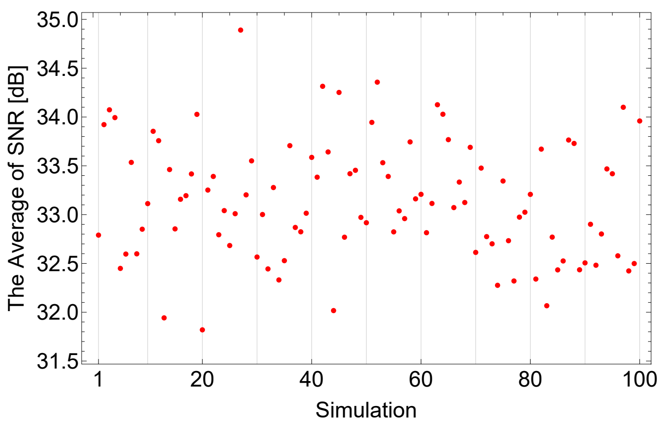

Fig. 11: The average of SNRs for each simulation

Table 8: A statistic for estimated mutual inductance $\tilde{M}$s

Average [μH]	Standard deviation [μH]	Minimum [μH]	Maximum [μH]
15.0	0.0124	14.8	15.5

Table 9: A statistic for the relative errors

Average %	Standard deviation %	Minimum %	Maximum %
0.322	0.826	-1.50	3.02

Table 10: A statistic for the avergae of SNRs

Average [dB]	Standard deviation [dB]	Minimum [dB]	Maximum [dB]
33.1	0.601	31.8	34.9

D. *Summary*

As the result of numerical simulations, if the system has a larger M, the relative errors tend to be smaller that we find in Table 3, Table 6, and Table 9. Additionally, the standard derivations of the estimated mutual inductance $\tilde{M}$s also tend to be smaller. These numerical simulation results show that if the system has a larger M, we can measure correctly the mutual inductance each measurement.

For the results of all mutual inductances, we plot a relationship between the average of SNR and relative error in Fig. 12.

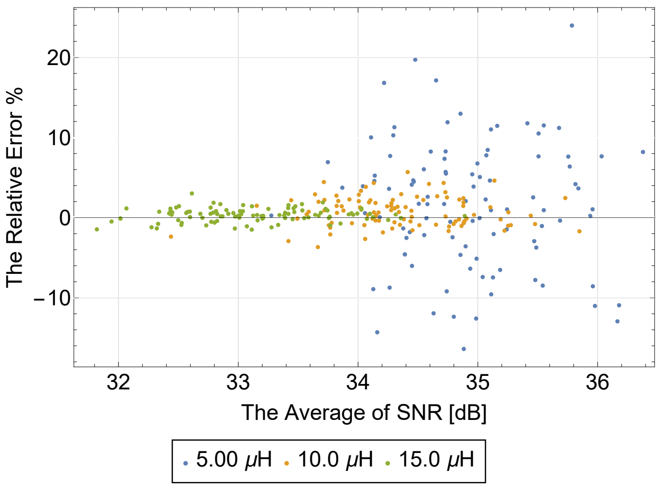

Fig. 12: The average of SNR versus the relative error for each mutual inductance estimation

Fig. 12 shows also that if the system has larger a M, the relative errors tend to be smaller , while if the system has a smaller M, the relative errors tend to be larger even if the average of SNR is larger, especially the case of $M = 5.00$ [μH]. The higher SNR means that the signal for estimating the mutual inductance is less affectable by the noise, and therefore the estimation performance will be better. However, in the case of mutual inductance $M = 5.00$ [μH], while the average of SNR is higher than the others, the estimation performance is worse. To clarify the relationship the estimation performance and the average of SNR in detail, in the next Scenario 2, we verify the relationship for the worst estimation case, $M = 5.00$ [μH].

VI. Verification Result of Scenario 2

In order to change the average of SNR, we can make the voltage amplitude of voltage source $|\hat{u}(j\omega)|$ higher or make the σ smaller. We note that σ means a standard derivation of measurement noise in the measurement device. Thus, we perform two numerical simulations with varying $|\hat{u}(j\omega)|$ or σ for the case of $M = 5.00$ [μH].

A. *In the case of varying $|\hat{u}(j\omega)|$*

We fix $\sigma = 0.08$ [A] and increase $|\hat{u}(j\omega)|$ from 20 to 70 [V] with 10 [V] step, and the result of this case is shown in Fig. 13.

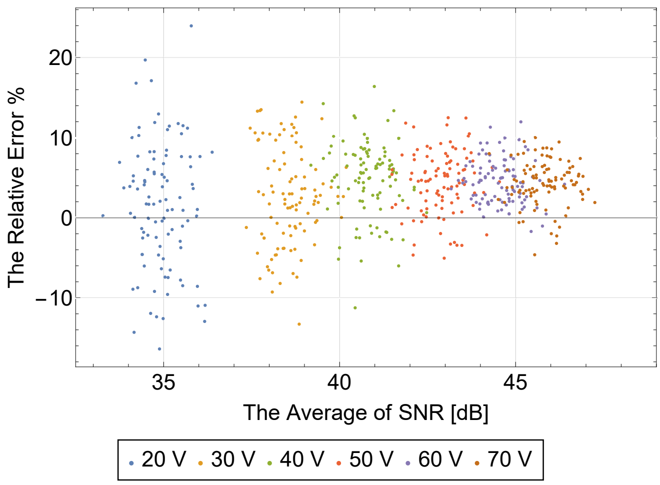

Fig. 13: The average of SNR versus the relative error by changing the voltage of voltage source $|\hat{u}(j\omega)|$

B. *In the case of varying σ*

We fix $|\hat{u}(j\omega)| = 20$ [V] and increase σ from 0.02 to 0.08 with 0.01 [A] step, and the result of this case is shown in Fig. 14.

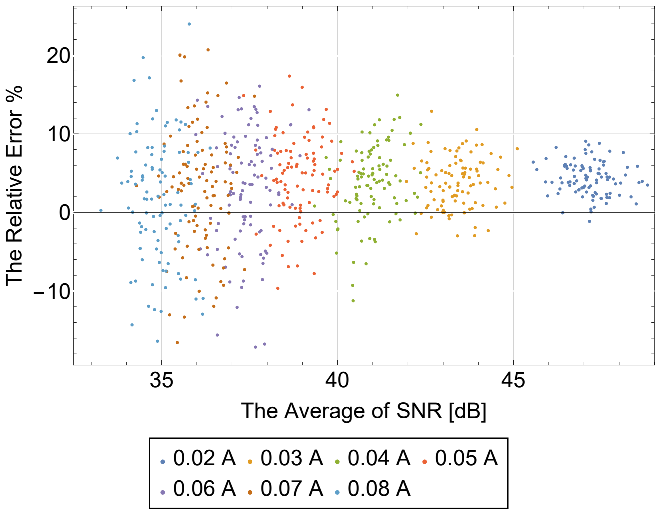

Fig. 14: The average of SNR versus the relative error by changing σ

C. Summary

From Fig. 13, we find that a larger $|\hat{u}(j\omega)|$ indicates that the average of SNR becomes higher, and the relative error becomes smaller. We find that a smaller σ also indicates that the average of SNR becomes higher, and the relative error becomes smaller from Fig. 14. These results mean that we can improve our estimation performance with adjusting these factors even if M is small.

VII. Discussion

In the Scenario 1, we performed the estimation under the fixed voltage of the voltage source $|\hat{u}(j\omega)| = 20$ [V], and the estimation performance was worse if the mutual inductance of the system is smaller (Table 3, Table 6, and Table 9). The smaller mutual inductance means that the magnetic coupling between the transmitter side and the receiver side is weak, and it will become a standalone circuit each side. Thus, the effects on the voltage and current by the transmitting circuit in the receiving circuit will be reduced and vice versa. As a result, the voltage and current do not have enough information for estimating the mutual inductance, and it may cause that the estimation performance is worse.

In the Scenario 2, we changed the voltage of the voltage source $|\hat{u}(j\omega)|$ from 20 V to 70 V with 10 V step. The estimation performance is better if the voltage of the voltage source is higher. The higher voltage of the voltage source makes the effects we mentioned the above higher, and therefore the voltage and current have enough information for estimating the mutual inductance, and it may cause that the estimation performance is better.

VIII. Conclusion

This paper has proposed a practical method for estimating mutual inductance in a WPT system. Our method has estimated correctly the mutual inductance in a relative error and standard deviation, if the mutual inductance is larger. On the other hand, estimation performance has been worse for a smaller mutual inductance. This result can be explained by that our method is based on an assumption that the WPT system has a magnetic coupling between its transmitter side and receiver side. Since the smaller mutual inductance M of the system means that the magnetic coupling will be weak, it will be approaching out of the assumption.

However, the estimation result can be improved by using the higher amplitude voltage of voltage source or current measurement device which has lower measurement noise. Thus, these factors should be considered as a system design process.

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